SIMULATION DUAL LOAD FWD SYSTEMS FOR DETERMINATION OF PAVEMENT LAYER MODULI IN FLEXIBLE PAVEMENT

BY

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TO my mother for her love

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The primary objective of this research was to develop layer moduli prediction procedures using a dual-load-rigid-plate Falling Weight Deflectometer (FWD). The investigation focused on the analyses of computer simulated pavement deflection response and the optimization of sensor positions.

Two different dual-load (24 and 40 inches) spacings and one adaptation of the conventional FWD system were evaluated in this study. The analytical sensor positions were chosen by performing sensitivity analyses to determine the effect of each of the layer moduli and their thicknesses. The flexible pavement structure was modeled as a four-layer system. A wide range of deflection basins, generated by the computer program (RIGID), were used for development of tentative layer moduli prediction equations.

Analytical evaluation of the dual-load FWD system indicated that double bending provides more information to discriminate the individual effects of all pavement layer moduli than the existing single-load FWD. For the dual-load-rigid-plate FWD with 24-inch load spacing, the moduli for each of the four layers can be predicted based on pavement thicknesses and deflection basin characteristics from the unique load sensor configuration. Generally, these predictive equations provide fairly reliable results. However, substantial prediction errors may occur when E_2 values are low. Therefore, the reliability of E_2 predictions should be checked using iteration or available back calculation techniques.

The analyses of the dual-load-rigid-plate FWD system with 40-inch load spacing resulted in the development of prediction equations for the asphalt concrete modulus (E_1), composite modulus (E_{12}), and subgrade modulus (E_4). This wider spacing provided better discrimination of E_1 and E_{12} for thick and very stiff pavements. However, it reduced the effect of subbase course modulus on the surface deflection and resulted in a significant interacting influence with other layer moduli. No relationships were identified for subbase moduli predictions.

The adaptation of the conventional FWD system using superposition to simulate a dual-load-rigid-plate system was found to be a very useful method even though measurement error may be magnified in the analysis.

CHAPTER 1
INTRODUCTION

1.1 Research Need

In highway engineering, accurate values of elastic moduli are needed in order to determine the allowable loads for existing pavement structures and the required overlay thicknesses, and to assess other rehabilitation needs. Elastic moduli for pavement systems typically are estimated *in situ* by the deflection measurements of NDT devices such as the Dynaflect and the Falling Weight Deflectometer. The various estimation procedures being used often involve (a) estimating the initial pavement moduli based on the expertise or experience of the individual; (b) predicting the deflection basin using an elastic layer computer program; (c) comparing predicted and measured deflection basins; (d) adjusting layer moduli to reduce differences between measured and predicted displacements; and (e) repeating (a) to (d) using updated moduli until the error between the two deflection basins are within allowable limits. This iteration procedure is often referred to as "backcalculation." Recent research has indicated that these analytical techniques have major drawbacks such as nonuniqueness in determining the moduli from the same deflection basin (1, 2, 3). It seems, therefore,

that a fast, economical and accurate method of determining *in situ* elastic moduli of pavement systems is needed.

1.2 Background and Objectives

Analytical work conducted by Ruth et al. (4) indicated that the existing single-load falling weight deflectometer (FWD) is incapable of discriminating among near surface layer moduli of flexible pavement systems. Comprehensive analyses suggested that a dual-load FWD can be used to overcome this deficiency when the loads are spaced sufficiently apart so as to induce a convex curvature in the pavement surface deflections between the loads. According to Roque et al. (5), the optimized center to center spacing between the load plates should be 40 inches since it could provide layer discrimination for a wide range of pavements. Therefore, this research concentrated on the development of the pavement layer moduli using surface deflections obtained from this dual-load-rigid-plate FWD system with a 7.14-inch diameter load plate.

In addition, a dual-load-rigid-plate system was designed for the FDOT using a beam attached to the FWD. However, the center to center spacing of the load plates was only 24 inches because of the constraints posed by the FWD trailer and loading system. This conversion does not conform to that desired based on the analytical studies performed by Roque. As an interim measure, however, it was anticipated that this dual-load-rigid-plate FWD would be used for evaluation of test

pavements to demonstrate the advantages of the system. Therefore, it was necessary to develop better sensor spacings and appropriate prediction equations.

The primary objectives of this research were

1. To evaluate the FDOT modified FWD unit (24-inch C-C loading system, 7.14-inch diameter load plate) and develop layer moduli prediction equations based on a deflection database generated by the computer program (RIGID) (6).
2. To generate a deflection basin data bank over a great range of layer moduli and pavement thicknesses for the desired FWD unit(40-inch C-C loading system, 7.14-inch diameter load plate) by the computer program (RIGID), and develop/evaluate layer moduli prediction equations.
3. To develop an adaptation of the FWD systems using superposition to simulate the dual-load-rigid-plate FWD system (11.82 diameter load plate) and establish tentative layer moduli prediction equations.

1.3 Scope and Methodology

The research effort was directed toward the analytical investigation of two dual-load-rigid-plate (24 and 40-inch load spacing) and one adaptation of FWD systems using the linear elastic multilayer computer program RIGID to simulate deflection basins for four-layer flexible pavement systems with different combinations of layer moduli. The generated

database was used to develop layer moduli prediction equations.

The development of the tentative pavement layer moduli prediction equations was carried out in three steps. Step one included sensitivity analyses to assess the effect of layer moduli and layer thickness on the magnitude and shape of deflection basins. The results of this analyses provided a good indication of which deflection and/or deflection difference could be used to predict layer moduli and where the sensors should be positioned to achieve superior layer discrimination.

Step two emphasized the establishment of the tentative layer moduli prediction equations.

Step three involved the evaluation of the reliability of the tentative layer prediction equations developed in step two.

CHAPTER 2
PAVEMENT LAYER MODULI PREDICTION METHODS

2.1 Introduction

Pavement rehabilitation work is now of key importance to those industrialized countries that undertook the construction of modern highway networks 30 or more years ago. There is, therefore, an immediate need to develop efficient and economical methods for the selection of maintenance and rehabilitation strategies at the network and project levels. This requires the determination of the residual life of pavement sections and pavement rehabilitation overlay design requisite.

The various evaluation methods currently available are generally classified into two categories: empirical methods and mechanistic (analytical) methods (7). Many researchers agree (8-15) that mechanistic (analytical) methods have obvious advantages over empirical methods, which are based on the correlation between the maximum deflection under a load and pavement performance. The mechanistic method allows a rational evaluation of the mechanical properties of the materials in the pavement structure.

The basis of the most commonly used analytical methods for pavement evaluation and for overlay design is the linear

elastic layered theory. In this theory, only two properties for materials characterization are required to determine the stress, strain, and displacement pattern, i.e., modulus of elasticity and Poisson's ratio. Generally, changes in the values of Poisson's ratio, unlike the elastic modulus, have no significant effect on the subgrade strain and tensile strain at the bottom of asphalt layers. Therefore, the elastic modulus is a key input parameter for pavement analysis using the layer theory (16-20).

Many different methods are used for measuring the elastic modulus of paving materials. It is important, however, that the testing method is capable of reproducing the major in-situ conditions. For instance, temperature and frequency are primary factors in determining the modulus of asphalt, while for granular materials it depends on the density and moisture conditions because of the nonlinearity of the material. Therefore, proper procedures must be followed at all times to obtain reproducible results. Frequently, the testing procedures for determining the elastic modulus of pavement materials are categorized into three classifications: laboratory tests, destructive tests, and in situ nondestructive tests (21).

Many of the laboratory testing methods have been developed as research tools to investigate the fundamental behavior of materials and are consequently complex and expensive. For routine design purposes, simpler and more

cost-efficient methods are required in the interests of acceptability. In this respect, the indirect tensile test and the repeated load triaxial test (22) are the most popular. Repeated load triaxial tests are useful for soil and unbound materials, while the indirect tensile test is good for bound materials (both asphalt and cement bound). Monismith et al. (23) studied the various factors affecting laboratory determination of the moduli of pavement systems and concluded:

> It is extremely difficult to obtain the same conditions that exist in the road materials (moisture content, density, etc.) and same loadings (including loading history) in the laboratory as will be encountered in situ....Thus the best method of analysis would appear to determine an equivalent modulus which when substituted into expressions derived from the theory of elasticity, will give a reasonable estimate of the probable deformation. (23, p. 112)

Destructive field tests include static plate loading and the California Bearing Ratio (CBR) test. These tests require trenching and subsequent repair of the pavement. These test methods are usually time consuming and expensive; therefore, only a few locations can be tested.

In situ nondestructive test (NDT) methods, which have gained wide popularity over the last decades, are considered as the simplest and most time-efficient methods. The advantage of these (NDT) methods is their ability to collect data at many test locations within a short time span. Therefore, a great deal of research effort has been concentrated on their use for pavement evaluation.

2.2 Nondestructive Testing Equipment

Nondestructive testing (NDT) devices are quite useful for the evaluation of the structural behavior of asphalt pavements. The measurement of the surface deflection basin of a pavement provides essential information for the evaluation of the asphalt pavement's structural characteristics.

NDT equipment used to collect data on existing pavements is categorized into four types of loading systems:

1. Static deflection
2. Steady state deflection
3. Impulse load deflection
4. Wave propagation

Table 2.1 (24) summarizes the basic characteristics of each of the common and commercially used NDT devices. Although the general framework for each device may appear to be similar, a tremendous amount of difference exists between the devices. Differences can be found in almost every aspect of the procedure, such as principles of operation, the load actuator system, the magnitude of load, type of load transmission, and the method of recording data (25-35).

It is generally agreed (31,34) that the peak response from an FWD test is quite close to the response of a moving wheel load of the same magnitude. Therefore, FWD measured data can, in principle, be used to back calculate pavement layer properties related to the remaining pavement life.

Table 2.1 Characteristics of Commercially Available Nondestructive Tests (24)

Device Name	Principle of Operation	Load Actuator System	Min. Load	Max. Load	Static Weight on Plate	Type of Load Transmission	Method of Recording Data
Benkleman Beam (AASHTO)	Deflection Beam	Loaded Truck Axle	N/A	N/A	N/A	Truck Wheels	Manual
Deflection Beam (British)	Mechanized Deflection Beam	Loaded Truck Axle	N/A	N/A	N/A	Truck Wheels	Manual
La Croix Deflectograph	Steady State Vibratory	Moving Truck	Empty Truck Weight	Loaded Truck Weight	N/A	Truck Wheels	Printer or Automated
Dynaflect	Steady State Vibratory	Counter Rotating Masses	1,000	1,000	2,100	Two 16" Diam. Urethanes Coated Steel Wheels	Printer or Automated
Model 400 B Road Rater	Impulse	Hydraulic Actuated Masses	500	2,800	2,400	Two 4" by 7" Pads with 5.5" Center Gap[a]	Printer or Automated
Model 2000 Road Rater			1,100	5,500	3,800	Circular Plate 18" diam.[a]	
Model 2008 Road Rater			1,000	8,000	5,800		
KUAB 50 FWD	Impulse	Two Dropping Masses	1,500	12,000	?	Sectionalized Circular Plate 11.8" diam.[a]	Printer or Automated
KUAB 150 FWD			1,500	35,000	?		
Dynatest Model 8000 FWD		Dropping Masses	1,500	24,000	?	Circular Plate 11.8" diam.	Printer or Automated

[a] Plates of other diameter are also available

Table 2.1 (Continued)

Device Name	Type of Carriage	Type of Prime Mover	Basic Cost ($)	Contact Area (in^2)	Vibratory Freq. & Range	Deflection Measuring System	Number of Deflection Sensors[a]	Load Measuring System
Benkleman Beam (AASHTO)	N/A	N/A	1,000	N/A	N/A	Dial Indicator	1	None
Deflection Beam (British)	N/A	N/A	1,500	N/A	N/A	Dial Indicator	1	None
La Croix Deflectograph	Truck	None	166,500	N/A	N/A	Displacement Transducers	2 (one in each wheel path)	None
Dynaflect	Trailer	Tow Vehicle	22,185	~32	8 Hz	Velocity Transducers	5	None
Model 400 B Road Rater	Trailer	Tow Vehicle	30,580	56	5 Hz to 70 Hz	Velocity Transducers	4	Load Cell
Model 2000 Road Rater			40,800	254			4	
Model 2008 Road Rater			64,000	254			4	
KUAB 50 FWD	Trailer	Tow Vehicle	70,000	109	N/A	Seimic Deflection Transducers	5	Load Cell
KUAB 150 FWD			85,000	109			5	
Dynatest Model 8000 FWD	Trailer	Tow Vehicle	86,500	109	N/A	Velocity Transducers	7	Load Cell

[a] More sensors may be added as necessary

Analytical work conducted by Ruth et al. (4) and Badu-Tweneboah et al. (36) demonstrated that the deflection basin resulting from the single-load falling weight deflectometer (FWD) did not allow for accurate discrimination of different pavement layer moduli, particularly the moduli of near-surface layers. Comprehensive analyses suggested that the double-bending resulting from a dual-load system such as the Dynaflect, allowed for much better discrimination when appropriate deflection measurements were obtained using a modified sensor configuration (Figure 2.1), which defines deflection basins in both the longitudinal and transverse directions. In addition, the actual pressure distribution applied to the pavement through the FWD loading plate is unknown. For rigid pavement and flexible pavement at low temperatures (very stiff), the assumption of uniform pressure distribution may be satisfactory, but for flexible pavements at mild temperature, the pressure distribution may be far from uniform. Assuming a uniform pressure distribution to analyze deflections under a mild weather condition may result in serious errors in the predicted near surface layer moduli. Furthermore, it is unknown whether the energy absorbed by the membrane on the loading plate has caused significant discrepancies between the load measured above the plate and the actual load applied to the pavement.

It appears that these problem would be solved by a developing dual-load-rigid-plate FWD system. The dual load system would improve the discrimination of pavement layer

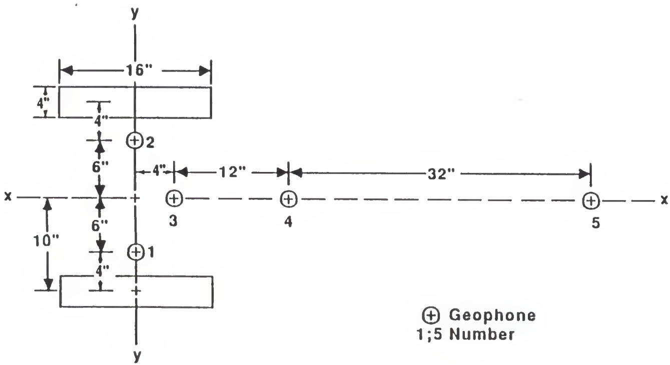


Figure 2.1 Modified Dynaflect Configuration

moduli and still maintaining the advantage of the original FWD system. The use of the rigid loads would reduce the uncertainty in the pressure distribution applied to the pavement and minimize or cancel the energy absorption. Research work done by Roque et al. (5) showed that the optimal center-to-center spacing between the load plates should be 40 inches. The diameter of the load plates that are conventionally used has no significant effect on the shape of the deflection basin.

2.3 Interpretation of the Pavement Deflection Basin

2.3.1 Principle of Interpretation

Once the deflections produced by NDT are measured, the remaining problem is to determine the set of layer moduli which correspond to that deflection basin. It is known that the thickness and moduli of the pavement layers and the subgrade modulus determine the shape of the deflection basin. Determining the moduli from the deflection basin therefore amounts to solving an inverse problem.

When a load is applied over a known area, generally, it is assumed that the load is distributed through the pavement system like the shape of a truncated cone. The zone of influence increases with increasing depth (represented by the dashed line in Figure 2.2) (37). At a sufficient distance from the load centerline, only the subgrade is stressed, and the deflection depends only on the subgrade modulus. At an

intermediate distance from the load centerline, the deflection depends on the subgrade modulus and on thicknesses and moduli of subbase and base course. At the load centerline, the deflection depends on the properties of the entire pavement structure.

Based on this concept, the properties of the subgrade are deduced from the deflection at an appropriate distance from the load; the properties of the subbase and base course are

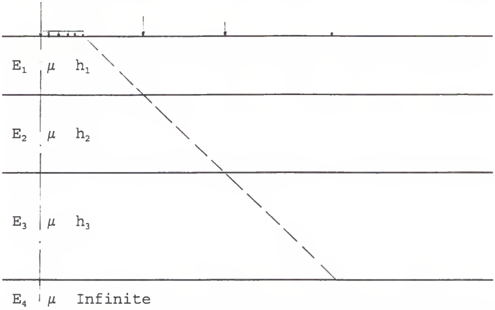


Figure 2.2 Four-Layer Elastic Representation of a Pavement System

related to the shape of the deflection basin at intermediate points; the asphalt concrete layer is related to the shape of the deflection basin near the load centerline (Figure 2.2) (37, 38). Consequently there are two ways to evaluate the properties of the pavement structure, either by empirical method or mechanistic method.

2.3.2 Empirical Methods

Empirical procedures directly relate NDT response parameters to the structural capacity of a pavement. Most of these methods (39) do not involve direct or indirect theoretical analysis. Instead, they are based on the correlation between the maximum deflection under a load (static NDT or wheel load) and pavement performance. Figure 2.3 shows an example of basin parameters and criteria used to evaluate a pavement (40). Table 2.2 lists some of the deflection basin parameters that have been developed for NDT data evaluation of pavements (41, 42).

Another possible empirical procedure is to compare the measured deflection basin of an existing pavement with the computed deflection bowl (based on a multi-layer elastic model) of a new pavement of the same composition, in which the materials have their normal values. The differences between the two deflection basin can be used to identify any structurally deficient layers in the existing pavement (42).

2.3.3 Mechanistic Method

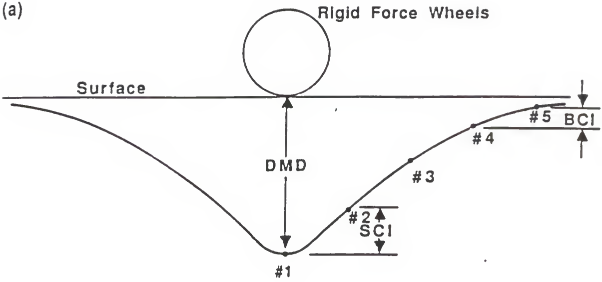
Mechanistic interpretation of NDT data is usually performed by one of the following:

1. Direct relationship between deflection parameters and the elastic moduli of the pavement layers

2. Back calculation of the moduli by fitting a measured deflection basin to a deflection basin using an iterative procedure

3. A combination of 1 and 2

(a)



DMD = Dynaflect Maximum Deflection (Numerical Value of Sensor No. 1)
SCI = Surface Curvature Index (Numerical Difference of Sensor No. 1 and No. 2)
BCI = Base Curvature Index (Numerical Difference of Sensor No. 4 and No. 5).

a) Basin Parameters

(b)

DMD	SCI	BCI	CONDITION OF PAVEMENT STRUCTURE
GT 1.25	GT 0.48	GT 0.11	Pavement and Subgrade Weak
		LT 0.11	Subgrade Strong, Pavement Weak
	LT 0.48	GT 0.11	Subgrade Weak, Pavement Marginal
		LT 0.11	DMD High, Structure Ok
LT 1.25	GT 0.48	GT 0.11	Structure Marginal, DMD Ok
		LT 0.11	Pavement Weak, DMD Ok
	LT 0.48	GT 0.11	Subgrade Weak, DMD Ok
		LT 0.11	Pavement and Subgrade Strong

b) Criteria

Figure 2.3 Empirical Interpretation of Dynaflect Deflection Basin (40)

Table 2.2 Summary of Deflection Basin Parameters (40,41)

Parameter	Definition[a]	NDT Device[b]
Dynaflect maximum deflection (DMD)	$DMD = d_1$	Dynaflect
Surface curvature index (SCI)	$SCI = d_1 - d_2$	Dynaflect, Road Rater model 400
Base curvature index (BCI)	$BCI = d_4 - d_5$	Dynaflect
Spreadability (SP)	$SP = (\Sigma d_i / 5d_1) \times 100$, $i = 1$ to 5	Dynaflect
	$SP = (\Sigma d_i / 5d_1) \times 100$, $i = 1$ to 4	Road Rater model 2008
Basin slope (SLOP)	$SLOP = d_1 - d_5$	Dynaflect
Sensor 5 deflection (W_5)	$W_5 = d_5$	Dynaflect
Radius of curvature (R)	$R = r^2/[2d_m(d_m/d_r-1)]$	Benkelman beam
Deflection ratio (Q_r)	$Q_r = r/d_0$	FWD, Benkelman beam
Area, in inches (A)	$A = 6[1 + 2(d_2/d_1) + 2(d_3/d_1) + d_4/d_1)]$	Road Rater model 2008
Shape factors (F_1, F_2)	$F_1 = (d_1 - d_3)/d_2$	Road Rater model 2008
	$F_2 = (d_2 - d_4)/d_3$	
Tangent slope (TS)	$TS = (d_m - d_x)/x$	

[a] d = deflection; subscripts 1,2,3,4,5 = sensor locations; 0 = center of load; r=radial distance; m=maximum deflection; x = distance of tangent point from the point of maximum deflection.

[b] The NDT device for which the deflection parameter was originally defined

Direct methods

Two different methods are most widely used for indirect analytical solution. One method has been developed only for two-layer systems which usually involve graphical solutions or nomography, which in most cases only provides an estimate of the subgrade modulus (43-48).

The other method consists of generating a set of deflection bowls, using a theoretical pavement structure model (generally a multi-layer linear elastic model), for a large number of different combinations of pavement layer thickness, moduli and subgrade moduli, then using multiple regression analyses to establish relations between certain indices describing the deflection basin and the moduli that are to be determined. This establishes predictive equations, which can be of the form

$$E_i = k_1(d_j-d_k)^{k2}$$

where d_j-d_k is the difference in deflection between two well chosen points on the deflection basin and k_1 and k_2 are functions of elements (regression coefficient) of the structure. This method has been developed by some researchers (4, 49). Based on this method, they successfully established simple power law equations to predict layer moduli using a modified dual-load Dynaflect sensor configuration with D_1 through D_5 having coordinates corresponding to that of Figure 2.1. These equations are of the form

$$E_4 = 5.40(D_5)^{-1.0} \qquad \text{Eqn. 2.1}$$

$$E_3 = 8.745(D_3 - D_4)^{-1.0919} \quad \text{Eqn. 2.2}$$

$$E_{12} = 60.611(D_1 + D_2 - 2D_3)^{-0.831} \quad \text{Eqn. 2.3}$$

$$E_{12} = 59.174(D_1 + D_2 - 2D_3)^{-0.805} \quad \text{Eqn. 2.4}$$

where for equation 2.3,

$E_{12} = (E_1t_1 + E_2t_2)/(t_1 + t_2)$, and

for equation 2.4,

$$E_{12} = [(E_1^{1/3}t_1 + E_2^{1/3}t_2)/(t_1 + t_2)]^3$$

Although this was a viable approach, the load imparted by the Dynaflect did not produce deflections of sufficient magnitude on stiff pavement systems to allow for good discrimination of layer moduli. Therefore, the concept of a dual load Falling Weight Deflectometer (FWD) was proposed to achieve double bending similar to the Dynaflect but with greater deflections to minimize the influence of deflection sensor precision limitations.

Back-calculation methods

Back calculation of pavement layer moduli is usually based on the best fit of computed surface deflections with measured surface deflections. Initially, this procedure was developed using a trial-and error approach (50-57), as described in the following steps:

1. Use pavement thickness, the loading, and estimate or "guess" a set of pavement layer moduli to compute the deflection basin (usually using a multilayer elastic computer program).

2. Compare computed and measured deflections.

3. Adjust the initial estimates to improve the fit between the predicted and measured deflections
4. Repeat 1 to 3 using upgraded moduli until the difference between the computed and measured deflection basins fall within an acceptable range.

Because of the time demands (58) of the trial-and-error method, a number of computer programs have been developed to perform the iteration. Table 2.3 lists some of the self-iterative computer programs (4). These computer programs were mostly based upon linear elastic theory and employing different models, deflection-matching algorithms and tolerance levels (59, 60). However, none of these programs are guaranteed to give reasonable moduli values for every deflection basin measured.

The limitation of the iterative elastic layer back calculation program as mentioned above is that it requires the user to have starting values and ranges for the layer moduli, i.e., it is user dependent. Based on a recent study (61), two agencies using the same computer program derived quite different back calculation results for the same pavement section. Therefore, unique solutions can not be guaranteed since an infinite number of layer modulus combinations can provide essentially the same deflection basins. Also, most of the iterative programs yield questionable base course and subbase moduli. In some programs, adjustment of the field data is required in order to improve the solution (61).

Table 2.3 Summary of Computer Programs for Evaluation of Flexible Pavement Moduli for NDT Devices (4)

Name	Reference	Number of Layers	Theoretical Model Used For Analysis	Applicable NDT Device
*	Anani (63)	4 Layer	BISAR-Elastic	Road Rater 400
ISSEM4	Sharma and Stubstad(64)	4 Layer	ELSYM5-Elastic	FWD
CHEVDEF	Bush (56)	4[(a)] Layer	CHEVRON-Elastic	Road Rater 2008
OAF	Majidzadeh and Ilves (65)	3 or 4 Layer	ELSYM5-Elastic	Dynaflect FWD, or, Road Rater
ILLI-Pave	Hoffman and Thompson(66)	3 Layer	Finite Element	Road Rater 2008, or FWD
*	Tenison (54)	3 Layer	CHEVRON's N (Elastic)	Road Rater 2000
FPEDD1	Uddin et al. (41)	3 or 4 Layer	ELSYM5-Elastic	Dynaflect, FWD
BISDEF	Bush and Alexander (67)	4[(a)] Layer	BISAR-Elastic	Dynaflect, Road Rater, FWD, WES Vibrator
ELMOD	Ullidtz and Stubstad (68)	2, 3 or 4 Layer	MET-Boussinesq	FWD
IMD	Husain and George (69)	3 or 4 Layer	CHEVRON-Elastic	Dynaflect, but can be modified for FWD
DYNAMIC	Mamlouk (55)	4 Layer	Elasto-dynamic	Road Rater 400

* not known or available
[(a)] not to exceed number of deflections

These difficulties often prevent pavement engineers from using the more reasonable mechanistic approach and lead them back to the traditional empirical approach.

The combination of direct and backcalculation methods

The two different methods, i.e., direct method and backcalculation method, can be combined. The advantage is that the first set of moduli is obtained by predictive equations. This results in faster convergence and a unique solution that does not depend on the operator. This method has been used in the REDAPS (REhabilitation Design of Asphalt Pavement Systems) computer program to predict elastic layer moduli, which was developed by Ruth and Guan (61).

REDAPS is a computer program that provides a fairly comprehensive mechanistic method for the analysis and rehabilitation design of flexible pavements (Fig. 2.4). In this program, the modulus of the asphalt layer (E_1) is computed using the asphalt viscosity-temperature relationship. This relationship was found (4,5,) to be linear on a log scale conforming to a power law equation (62) as presented in equations 2.5 through 2.12.

1) Asphalt viscosity - temperature relationship

$$\log(\eta_{100}) = B_0 + B_1 \log(^\circ K) \qquad \text{Eqn. 2.5}$$

where η_{100} (in Pa.S) is the viscosity of the asphalt concrete layer at a constant power of 100 Watt/m^3, and mean pavement temperature in degree Kelvin.

Coefficients B_0 and B_1 should be determined by the Schweyer Rheometer tests described in reference 62.

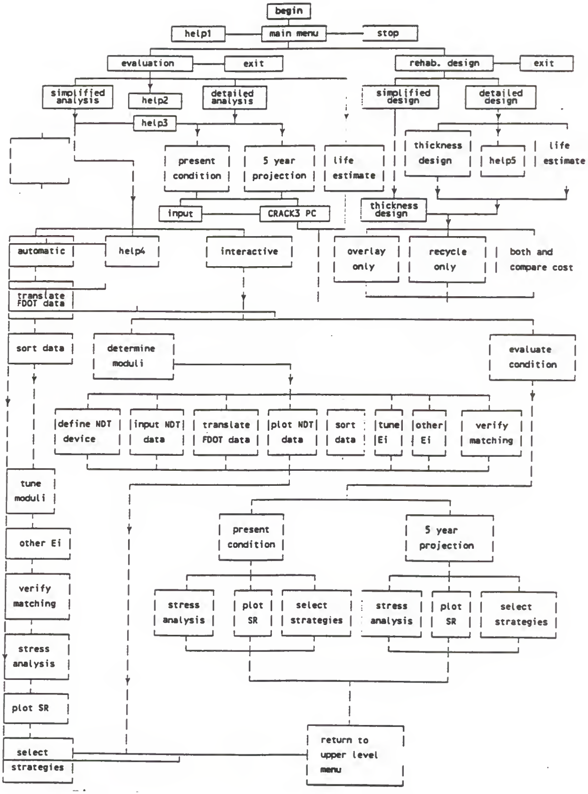


Figure 2.4 Flow Chart of the REDAPS Program

2) Asphalt concrete layer modulus

A) when the viscosity-temperature relationship is known

a) when $\eta_{100} \leq 1.87E4$ Pa.S

$$E_1 = 90 \text{ ksi} \qquad \text{Eqn. 2.6}$$

b) when $1.87E4 \leq \eta_{100} \leq 8.31E6$ Pa.S

$$\log(E_1) = 7.960 + 0.195 \log(\eta_{100}) \qquad \text{Eqn. 2.7}$$

c) when $8.31E6 \leq \eta_{100} \leq 9.19E8$ Pa.S

$$\log(E_1) = 7.18659 + 0.30677 \log(\eta_{100}) \qquad \text{Eqn. 2.8}$$

d) when $9.19E8 \leq \eta_{100} \leq 2.07E10$ Pa.S

$$\log(E_1) = 9.51354 + 0.04716 \log(\eta_{100}) \qquad \text{Eqn. 2.9}$$

e) when $\eta_{100} \geq 2.07E10$ Pa.S

$$E_1 = 1453 \text{ ksi} \qquad \text{Eqn. 2.10}$$

where E_1 (Pa) is the resilient modulus (sometimes referred to as dynamic modulus) of asphalt concrete, η_{100} (Pa.S) is determined by equation 3.6.

B) When the viscosity - temperature relation is unknown

a) if the pavement has no cracks

$$\log(E_1) = 6.4147 - 0.148 \text{ T} \qquad \text{Eqn. 2.11}$$

where E_1 is in psi and T in degree C.

b) if the pavement has considerable cracking (i.e., medium frequency class 2 and 3)

$$\log(E_1) = 6.4167 - 0.01106 \text{ T} \qquad \text{Eqn. 2.12}$$

Equations 2.11 and 2.12 are approximations applicable only for aged asphalt concrete pavements.

For other layers, moduli (E_2, E_3, E_4) are, initially, estimated from modified Dynaflect deflections by prediction

equations 2.1 through 2.4. Subsequently, these moduli are tuned to obtain the best fit within the measured deflections.

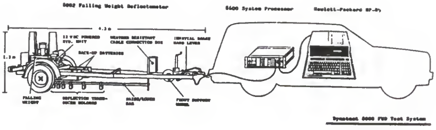
The REDAPS program can overcome the problem of arbitrary selection of initial moduli by utilizing the prediction equations. Also, the prediction equations were restricted by setting limits for the layer modulus equations (61) to eliminate unrealistic values. Therefore, this program can provide an effective pavement rehabilitation strategy.

CHAPTER 3
DESCRIPTION OF THE FALLING WEIGHT DEFLECTOMETER TESTING SYSTEM

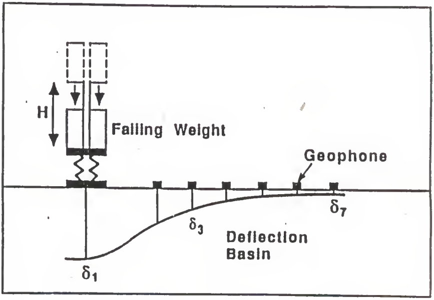
3.1 Introduction

The Falling Weight Deflectometer (FWD) is a deflection testing device operating on the impulse loading principle. This equipment uses a weight that is lifted to a given height on a guided system and is then dropped. By varying the mass of the falling weight, the drop height, or both, the impulse force can be varied. Figure 3.1 shows a schematic of the configuration and deflection basin (30).

The duration of the loading pulse is controlled by the buffer characteristics (Figure 3.1), and it closely approximates a half-sine wave. The duration of the load is between 25 and 30 milliseconds which is similar to the pavement response produced by a wheel load (31). Research by Hoffman and Thompson (31), Bohn, Ullidtz, Stubstad, and Sorenson (32), and Ertman-Larsen and Stubstad (33) indicated that the peak response from an FWD test is quite close to the response of a moving wheel load of the same magnitude. Figure 3.2 shows a comparison between FWD and moving wheel load response (33).



(a)



(b)

Figure 3.1 Schematic of FWD Load-Geophone Configuration and Deflection Basin (30)

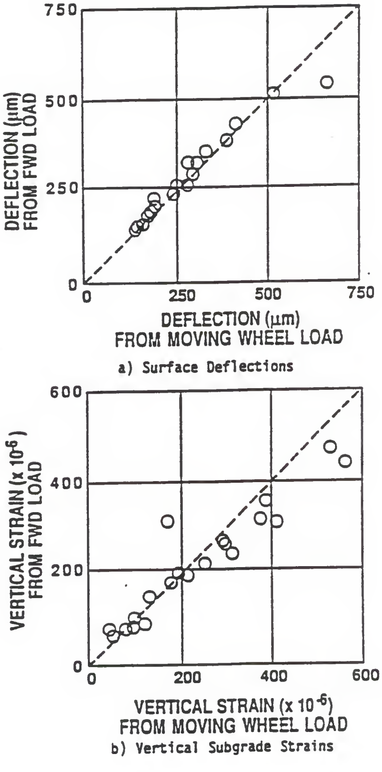


Figure 3.2 Comparison of Pavement Response from an FWD and Moving-Wheel Loads (33)

The response of the pavement to impulse loading is normally measured with a set of velocity transducers placed at different radial distances from the center of loading plate. These FWD measured data can, in principle, be used to back calculate pavement layer properties from which the remaining pavement life can be estimated.

3.2 Description of the Standard Falling Weight Deflectometer Testing System

The most widely used falling weight deflectometer (FWD) in the United States is the Dynatest model 8000 Falling Weight Deflectometer System. This type is also used by the Florida Department of Transportation (FDOT). The Dynatest FWD consists of a large mass that is constrained to fall vertically under gravity onto an 11.82-in diameter spring-loaded plate resting on the pavement surface. A force can be adjusted from 1,500 to 24,000 pounds by varying the drop heights and drop weights. The impulse load is measured by load transducer (load cell), while the deflection is measured by seven velocity transducers with one in the center of the loading plate. An CAMPAQ-286 computer controls the complete operation and records the information from the transducers and load cell on paper tape and a magnetic cassette.

3.3 Description of the Falling Weight Deflectometer Testing System Used in the Research

Instead of directly using the standard FWD system in this research, three modified FWD systems were used. One is a

dual-load-rigid-plate FWD system with 24-inch load spacing and 7.14-inch diameter load plate (Figure 3.3), one with 40-inch load spacing and 7.14-inch diameter plate (Figure 3.4), and one is adaptation of the standard FWD system with 11.82-inch diameter load plate (Figure 3.5) using superposition to simulate the dual-load-rigid-plate FWD system. The dual-load-rigid-plate FWD with 40-inch load spacing, which was recommended by Roque and Romero (5, 49), is intended to be part of a system to predict effective elastic layer moduli.

The dual-load-rigid-plate FWD system with 24-inch load spacing was designed for FDOT using a beam attached to the FWD model 8000. Because of the constraints posed by the FWD trailer and loading system, this conversion does not conform to the desired 40-inch spacing suggested with the analytical studies performed by Roque and Romero (5, 49). Like the standard FWD, the dual-load-rigid-plate system consists of a mass that falls vertically onto a buffer system. The difference is that the load is transmitted to two rigid plates, which are connected through a steel beam and rest on the pavement surface. The two loads produce a deflection basin with a convex shape (Figure 3.6). This shape will be shown to provide superior discrimination of the effects of individual layers on deflection response.

The established prediction equations from this study could not be evaluated by field tests because a suitable dual-load-rigid-plate FWD was not available. Therefore, an

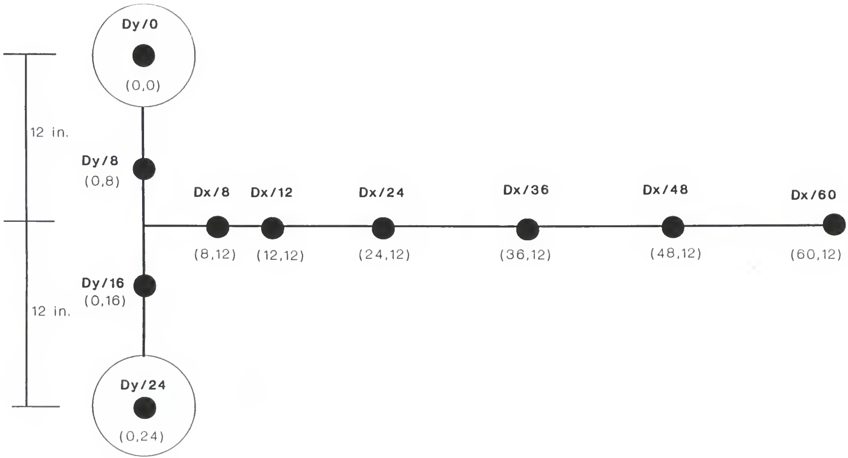


Figure 3.3 View of a Dual-Load-Rigid-Plate Falling Weight Deflectometer with 24-inch Spacing

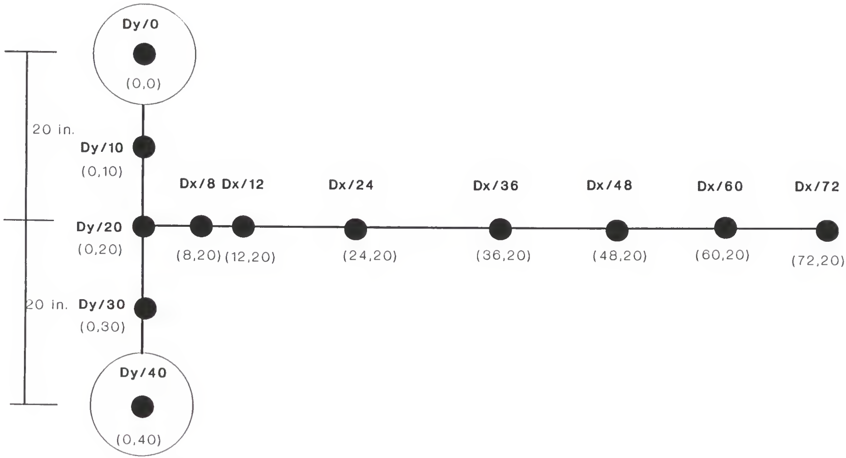


Figure 3.4 View of a Dual-Load-Rigid-Plate Falling Weight Deflectometer with 40-inch Spacing

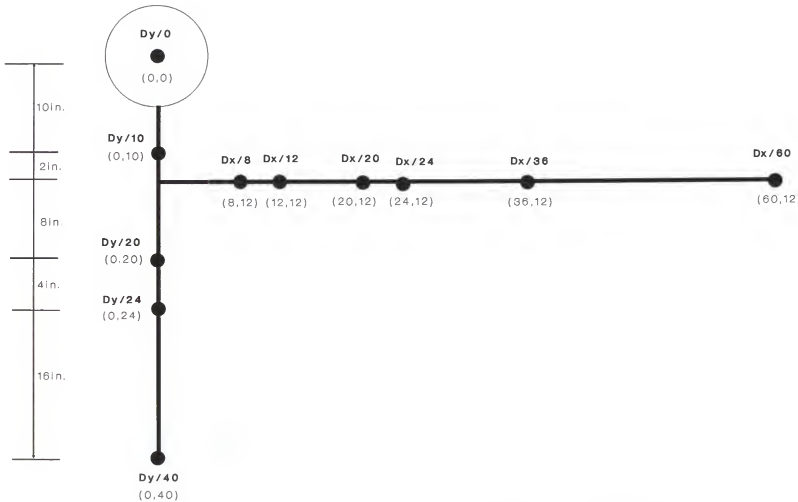


Figure 3.5 View of an Adaptation of Standard Falling Weight Deflectometer

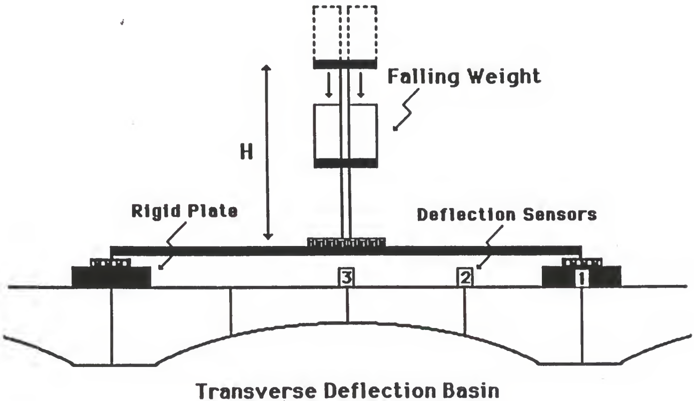


Figure 3.6 Schematic of a Dual-Load-Rigid-Plate FWD Load Sensor Configuration and Deflection Basin in the Transverse Direction

adaptation of the standard FWD system was designed to simulate the dual-load-rigid-plate FWD system. It is normally considered that the pavement structures are homogenous, linear and elastic. Therefore, single loads can be superposed to obtain dual load deflection basin. This theory was used in the adaptation of the standard FWD system.

CHAPTER 4
COMPUTER PROGRAM

4.1 BISAR Computer Program

The BISAR (Bitumen Structures Analysis in Roads) computer program was developed by Koninklijke/Shell Laboratorium, Amsterdam, Holland. The program is generally used to compute stresses, strains and displacements in elastic multilayered systems subjected to one or more uniform loads, acting uniformly over circular surface areas (53, 66). The surface loads can be combinations of a vertical normal stress and unidirectional tangential stress. The use of BISAR to compute the state of stress or strain in a pavement requires the following assumptions (4):

1. Pavement structure layers are horizontally continuous, isotropic, homogeneous, linearly elastic medium.
2. Each layer has finite thickness except for the lowest layer, and all are infinite in the horizontal direction.
3. The surface loading acts as a uniform distribution over a circular area.
4. The interface conditions between layers can be represented as either perfectly smooth (zero bond) or perfectly rough (complete bonding).
5. Inertia forces are negligible.
6. Deformations throughout the system are small.
7. The stress solutions are characterized by two material properties, Poisson's ratio and elastic modulus for each pavement layer.

The computer Program RIGID developed by Roque was based on the BISAR program. It is applicable to those surface loadings distributed under a rigid plate (nonuniform distribution).

4.2 RIGID Computer Program

The RIGID computer program was used to simulate rigid plate FWD deflection data to establish predictive moduli equations. This program was developed by Roque to determine the stress distribution under a rigid plate on a multilayer elastic system, and the deflection, stress, and strain response of the system subjected to the rigid plate load (5). In the program, the stress distribution under the rigid plate was approximated using a series of concentric rings of uniform pressure to produce the same deflections at all points under a circular loaded area. Existing elastic layer theory computer programs generally handle only uniformly loaded circular loaded area, so that circular rings must be modeled by subtracting the results (deflections, stresses, and strains) of a smaller circular load from those of a larger circular load. The effect (pavement response) of several contiguous concentric rings can then be added together to model rigid-plate (non-uniform) stress distributions.

The computer program BISAR was used to develop this approach. As shown in Figure 4.1, six uniformly loaded concentric rings were used to model the nonuniform stress

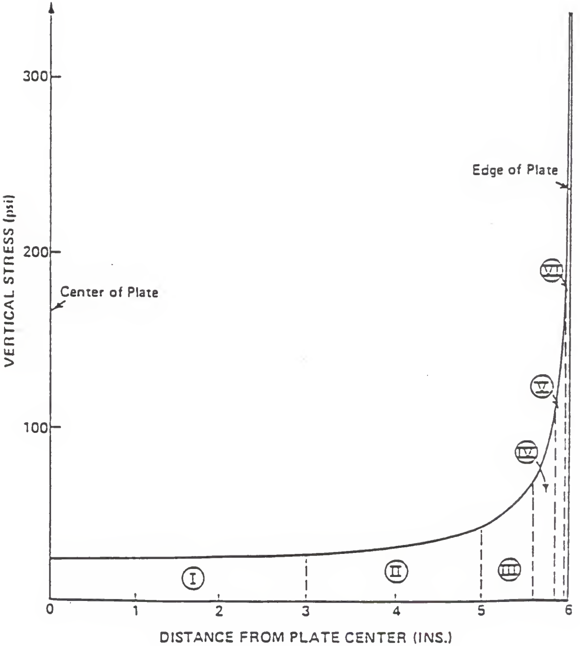


Figure 4.1 Stress Distribution under a Rigid Plate on a Semi-Infinite Mass: 50 psi Average Pressure

distribution under a circular area. An automatic adjustment procedure was devised and computerized to determine what combination of ring stresses (stress distribution) produced the same deformation (± .00001 in) under each ring for a particular combination of layer moduli. The iterative trial and error procedure adjusts the stresses based on the results of the solution for the previous set of stresses, until convergence is achieved. The total load is always kept constant. Results obtained using RIGID compared almost exactly with results obtained from closed-form solutions for rigid plate loadings on elastic materials.

CHAPTER 5
RESEARCH APPROACH

5.1 Introduction

Elastic layer theory computer programs are readily available and widely used and accepted to predict the stress, strain, and deformation response of the asphalt concrete in flexible pavement systems. The effective layer moduli are key parameters for use in these analysis programs. Most of the methods available for determining the elastic moduli of flexible pavements have been outlined in Chapter 2. These include the use of destructive and nondestructive tests. The limitations of these methods and the need for simpler approach have also been highlighted. An approach which develops a simplified method of predictive elastic layer moduli with the use of dual-load-rigid-plate FWD systems are therefore emphasized in this study. This approach included three dual-load-rigid-plate FWD systems. One is dual-load-rigid-plate FWD system with 24-inch load spacing (dual-load-rigid-plate FWD-24), one with 40-inch load spacing (dual-load-rigid-plate FWD-40), and one is an adaptation of the standard FWD system using superposition to simulate the dual-load-rigid-plate FWD system. The study focused on the development of moduli prediction equation based on computer program modeling of FWD deflection data.

5.2 Description of Approach

This research emphasizes the establishment of a set of simplified layer moduli prediction equation for the new dual-load Falling Weight Deflectometer and corresponding sensor configuration. These equations can avoid the need for the user to arbitrarily select a set of initial layer moduli for back calculation purposes, which increase the reliability of the predicted pavement moduli, and therefore, increase the accuracy of the mechanistic method and design process. This study consists of analyses for three different FWD configuration. One study established prediction equations for dual-load-rigid-plate FWD-24, one for adaptation of standard FWD system, and one for dual-load-rigid-plate FWD-40. The research approach for the three systems are shown in the flow charts of Figures 5.1 and 5.2, respectively. These two flow charts are very similar except that the chart for adaptation of standard FWD system excludes sensitivity analysis. The 40-inch load spacing was suggested by Roque and Romero (5, 49). But this equipment does not exist at this time. Therefore, field testing cannot be conducted. The study of the dual-load-rigid-plate FWD-40 and FWD-24 system were only intended to develop tentative prediction equations and to show how this system works. For the adaptation FWD system, sensor positions were selected from the analytical study on the basis of being used to predict the moduli of specific layers for the two dual-load-rigid-plate FWD systems. Field testing will be

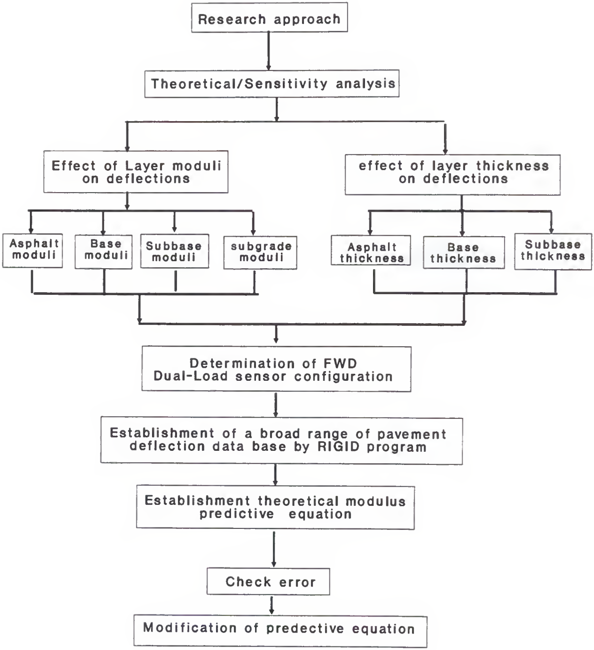


Figure 5.1 Research Approach Flowchart for Dual-Load-Rigid-Plate FWD-24 and FWD-40 Systems

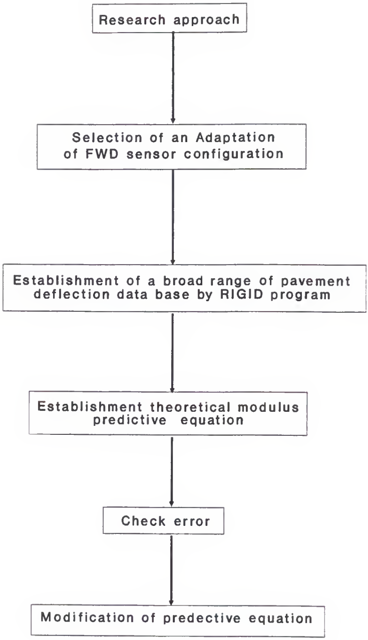


Figure 5.2 Research Approach Flowchart for Adaptation FWD System

performed in near future. A description of the flow charts is presented below.

The layer moduli prediction equations were developed by performing a sensitivity analyses to assess the effect of layer moduli and thicknesses on deflection basins. The flexible pavement structures were modeled as a four-layer system. The dual-load-rigid-plate FWD sensor configurations can be simulated by the RIGID program that predicts surface deflection data for four-layer pavement systems.

The sensitivity analyses included the determination of the effects that each of the layer moduli and their thicknesses have on the deflection basins. This initial analysis determined which deflections are most sensitive to specific layer moduli and thicknesses. These initial findings gave a good indication of how sensitive the systems were and where the sensors should be positioned to achieve effective layer discrimination. As a result, optimum sensor configurations for the dual-load-rigid-plate FWD systems were established and used to generate the deflection database for the development of predictive layer moduli equations.

In addition, the sensitivity analysis can also provide information as to which deflections and/or deflection differentials are related to the specific layer modulus and thickness. These findings can be used as the basis for development of comprehensive equations to predict layer moduli for a broad range of pavement geometries and layer moduli.

Tables 5.1 to 5.3 give the range of values considered in the analyses for the dual-load-rigid-plate FWD-24 system, for an adaptation of the standard FWD system, and for the dual-rigid-load FWD-40 system, respectively. The RIGID program was used to generate the data bank for each FWD system with the range of variables as listed in these Tables.

The selection of layer moduli and thicknesses, as given in Tables 5.1 and 5.2, was based on typical ranges in parameters representative of Florida's flexible pavement systems. In general, the stabilized subgrade thickness was fixed at 12 inches. Table 5.3 represented the typical ranges of general flexible pavement structures.

The surface deflection databases were established using the RIGID program for all possible combinations of layer moduli and pavement thickness as defined in Tables 5.1 through 5.3. In this study, the deflections predicted by the program were referred to as the "actual deflections, and the moduli used as input were referred to as the "actual moduli". From the actual deflections, a set of regression equations was developed to estimate the actual moduli.

Once the regression equations were developed, the reliability of the relationships was determined by comparing the actual to the predicted moduli. In this process, the actual moduli were those moduli used to get "actual

deflections" by RIGID program. The predicted moduli were obtained from prediction equations using "actual deflections". Then, the residue being the difference between the actual and the predicted modulus was determined. By comparison of the residue with layer thickness and/or other deflection variable, the true functional relationship might be approximated. As a result, the new regression model might be determined and the prediction equations be modified.

Table 5.1 Moduli and Thicknesses Used in the 24-in Spacing FWD Database

Layer	Modulus (ksi)	Thickness (in)
Asphalt Concrete	150, 300, 600, 1200	3, 6, 8
Base Course	42.5, 85, 120	8, 12
Subbase	15, 45, 75	12
Subgrade	5, 20, 40	infinite

Table 5.2 Moduli and Thicknesses Used in the adaptation of Standard FWD Database

Layer	Modulus (ksi)	Thickness (in)
Asphalt Concrete	150, 300, 600, 1200	3, 6, 8
Base Course	42.5, 85, 120	8, 12, 16
Subbase	15, 45, 75	12
Subgrade	5, 10, 20, 40	infinite

Table 5.3 Moduli and Thicknesses Used in the 40-in Spacing FWD Database

Layer	Modulus (ksi)	Thickness(in)
Asphalt Concrete	200, 500, 800, 1200	3, 6, 9, 12
Base Course	20, 40, 85, 120	5, 10, 20
Subbase	10, 40, 80 **	5, 10, 15
Subgrade	5, 10, 20, 35, 50	infinite

** The subbase modulus was varied between the subgrade modulus and the base modulus (i.e. it was never allowed to be greater than the base modulus or less than the subgrade modulus).

CHAPTER 6
PREDICTION EQUATIONS FOR FWD WITH 24-INCH LOAD SPACING

6.1 Introduction

The FWD with 24-inch load spacing with various configurations was simulated in the RIGID plate approximation program to predict deflection values for four-layer pavement systems. In these analyses, the deflections predicted by the program were referred to as the "actual deflections", and the moduli used as input were referred to as the " actual moduli". The "actual" variables were used to develop a set of regression equations. Thus, the moduli determined by the prediction equations were the "predicted moduli", and the deflection obtained from the computer program using the "predicted moduli" were the "predicted deflections". The goodness of fit for the regression models was determined by comparing the actual moduli with the predicted values.

6.2 Sensitivity Analysis of Theoretical FWD Deflection Basins

6.2.1 Parametric Study

Analyses were conducted to determine the sensitivity of the dual-load-rigid-plate system with the 24-inch spacing. This was accomplished by using the pavement section shown in Figure 2.2 as a typical Florida pavement under warm

temperature conditions. Using the information in Figure 2.2, each layer modulus or thickness was varied while the others were kept constant. For example, the modulus of the asphalt concrete changed from 200 ksi to 1200 ksi without changing the base, subbase, and subgrade moduli and the layer thicknesses. The load used in the sensitivity study was 8,000 pounds. The RIGID program was then used to calculate the deflection basins. The basins gave a good indication of how sensitive the system was and where the sensors should be positioned to achieve superior layer discrimination.

6.2.2 Variations in Pavement Parameters

Figure 6.1 shows the effect of change in asphalt concrete modulus on the deflection basins. It can been seen that the shape of the transverse deflection basin was affected by the modulus of the asphalt concrete layer. The asphalt concrete modulus had no apparent effect on the deflections in the longitudinal direction at offset distance over 24 inches. A change in layer modulus was mainly reflected in the deflections under the center of the load. The further away the produced deflection is from the load, the less the effect of the asphalt concrete modulus. Similar analyses for other layer moduli and thicknesses clearly indicated that only asphalt concrete layer thickness had an effect on the shape of the transverse deflection basin. Therefore, a relationship for asphalt concrete layer modulus could be established using the deflection difference in the transverse direction and asphalt layer thickness as key parameters.

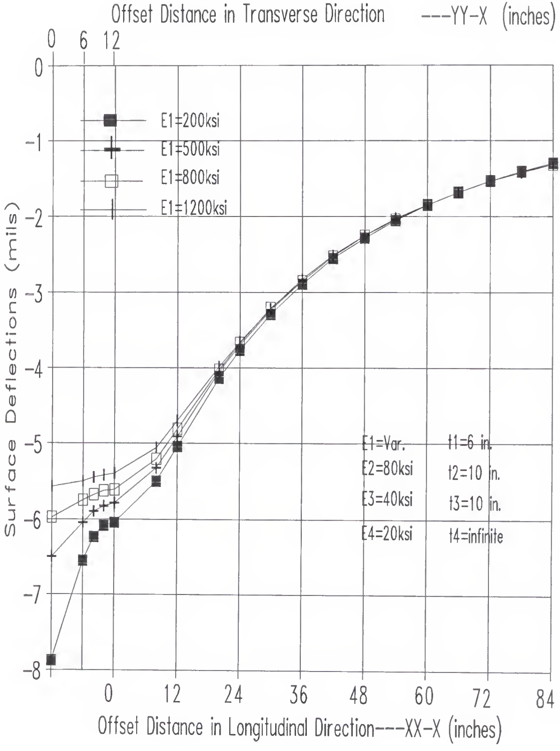


Figure 6.1 Effect of Variable Asphalt Concrete Layer Moduli on Surface Deflections

Figure 6.2 shows the effect of change in base course modulus on the deflection basins. It was observed that the base modulus had little influence on the shape of the transverse deflection basin. However, it had a very predictable effect on the longitudinal deflections closer to the loads. The most sizeable change occurred at a zero offset distance along the longitudinal direction. Beyond 30 inches, the change in deflections was almost zero. Similar analysis for other layer moduli and thicknesses indicated that asphalt concrete layer modulus and asphalt layer and base course layer thickness (t_1 and t_2) also had an effect on the shape of deflection basins in the longitudinal direction close to the loads. This appeared to indicate that the base modulus was not the only parameter relating to that portion of the longitudinal deflection basin close to the applied loads. Therefore, a combined modulus for E_1 and E_2 (composite modulus E_{12}) was used to establish a relationship with deflection difference between $D_{y/0}$ and $D_{x/12}$ or $D_{y/0}$ and $D_{x/8}$.

Similar sensitivity analyses were used for the subbase as for asphalt concrete and base modulus, which is shown in Figure 6.3. The transverse deflection basins remained nearly parallel to each other for different subbase moduli, indicating that the subbase modulus had no effect on the shape of the transverse deflection basins. The change in subbase modulus produced only a shift in the magnitude of transverse deflections. In the longitudinal direction, the changes in the deflections were observed only up to about 48 inches along

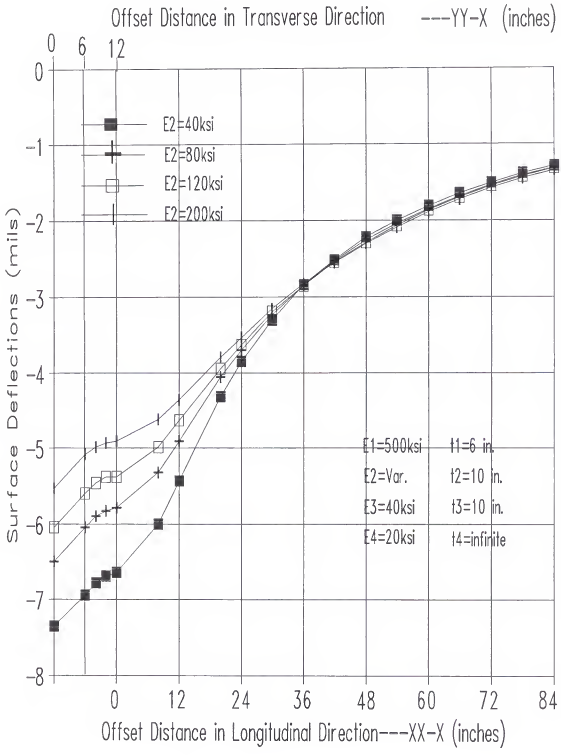


Figure 6.2 Effect of Variable Base Course Moduli on Surface Deflections

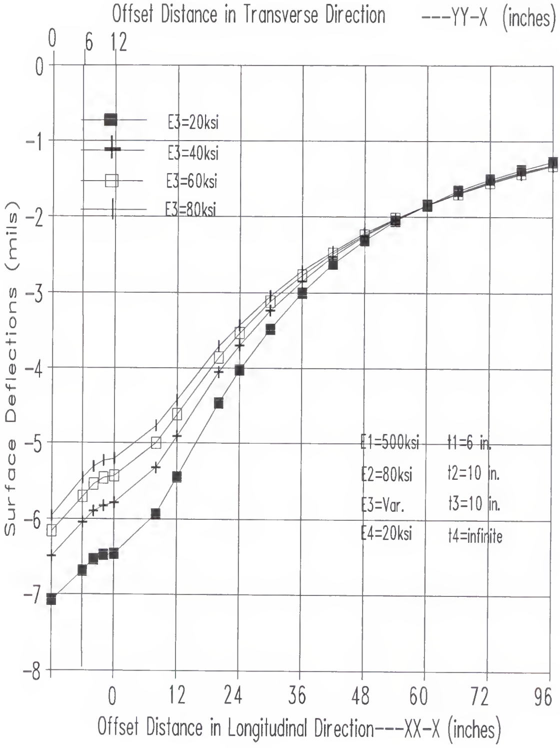


Figure 6.3 Effect of Variable Subbase Course on Surface Deflections

the centerline. Beyond this distance, the changes of deflections approaches zero. It was inferred that the subbase could be related to that portion of the longitudinal deflection basin further away from the loads than for the base course.

The changes in the subgrade moduli, as shown in Figure 6.4, indicated that the effect of the subgrade modulus was primarily on the magnitude of the deflections. The shape of the transverse deflection basin was the same for the different subgrade modulus. In the longitudinal direction, the subgrade modulus still had a considerable effect on the deflection at a distance of almost 60 inches, while other layer moduli had no discernable effect at this distance. Therefore, the deflection at the distance of 60 inches could be uniquely related to the subgrade modulus.

Similarly, a sensitivity study was also performed to assess the effects of layer thicknesses on the deflection basins. Figures 6.5 through 6.7 show the results of the analyses. According to these Figures, asphalt and base course layer thicknesses seemed to have the major effect. The effect of subbase layer thickness on the deflection shape along the transverse direction appeared to be small. Generally speaking, a change in layer thickness was reflected in the same way as a change in modulus.

These sensitivity analyses indicated that specific deflections or deflection differentials from a dual-load system could be related to the stiffness (modulus and thickness) of specific pavement layers. Accordingly

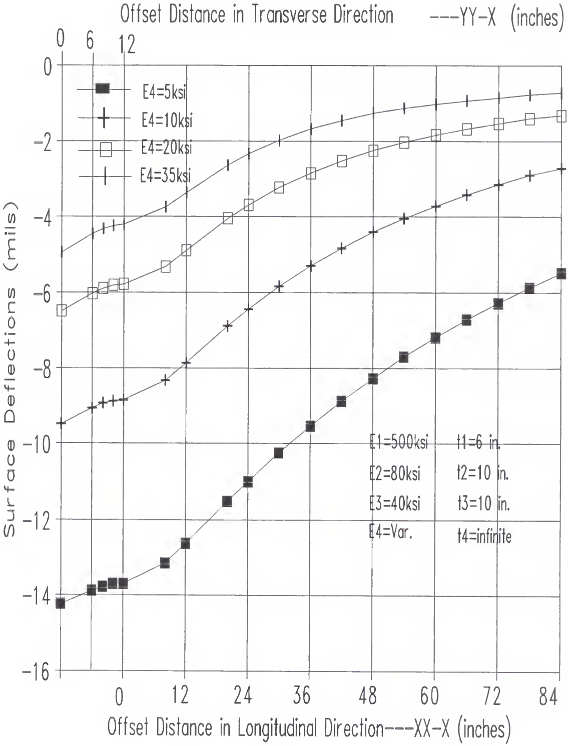


Figure 6.4 Effect of Variable Subgrade Moduli on Surface Deflections

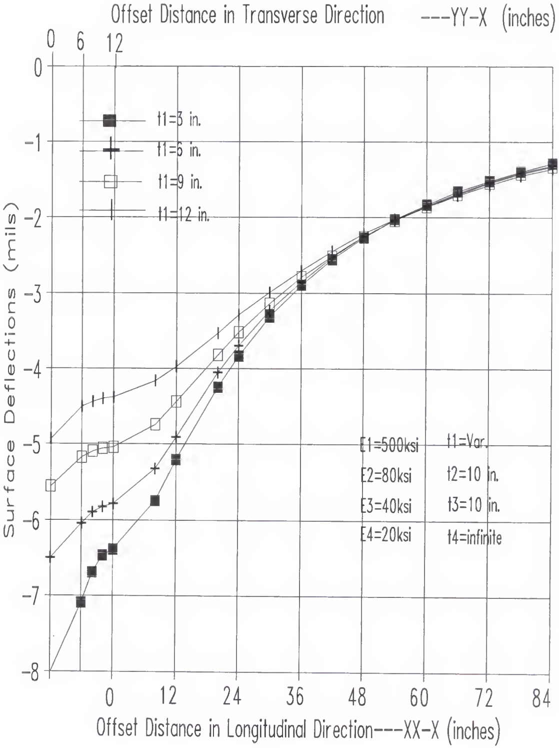


Figure 6.5 Effect of Variable Asphalt Concrete Layer Thickness on Surface Deflections

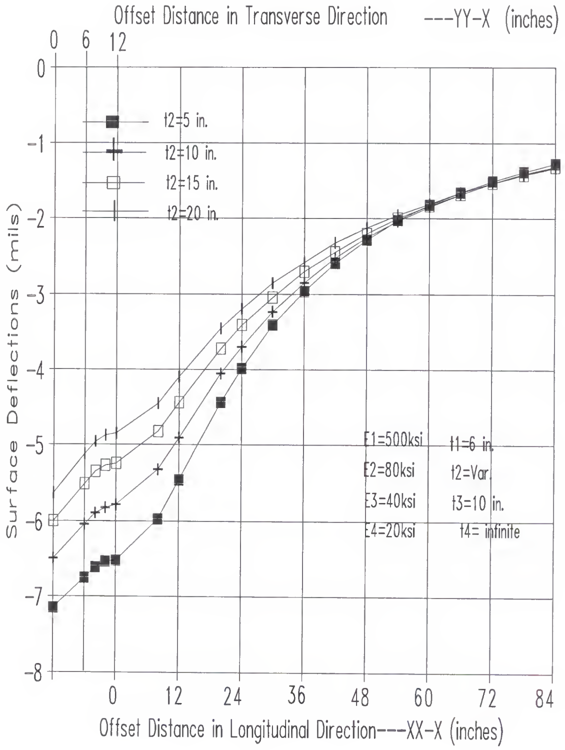


Figure 6.6 Effect of Variable Base Course Thickness on Surface Deflections

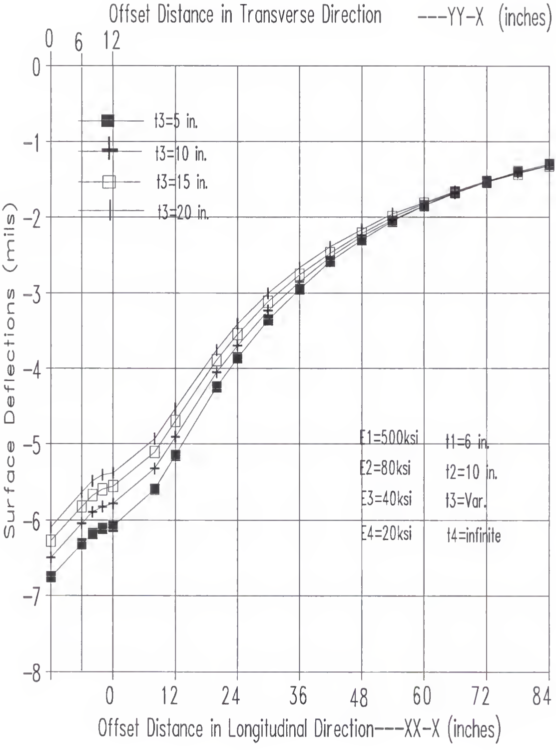


Figure 6.7 Effect of Variable Subbase Course Thickness on Surface Deflections

a set of comprehensive equations were developed to predict layer modulus for the range given in Table 5.1.

6.3 Development of Layer Moduli Prediction Equations

The elastic-layer computer program RIGID was used to generate FWD deflection basins for the 24-in dual-load-rigid-plate configuration using different combination of moduli and thicknesses covering the range listed in Table 5.1. These deflection data were evaluated to determine if the deflection response from one or more geophone positions could provide a unique relationship for prediction of individual layer moduli.

The development of layer moduli prediction equations was done by three steps (70, 71):

Step 1: Identify those variables which can be used to predict layer moduli.

Step 2: Ascertain the nature of the relationship between those chosen variable(s) and the layer moduli.

Step 3: Check whether the assumptions underlying the chosen model are valid.

6.3.1 Selecting the Variables (**Step 1**)

Trying to select reasonable candidate independent variables for inclusion in a regression model was a difficult task. In this study, this step was accomplished by performing the parameters sensitivity analysis as described above.

The sensitivity analysis had indicated that the difference between $D_{y/0}$ and $D_{x/8}$ deflection essentially

eliminated the effect of the underlying layers and was primarily dependent upon the moduli and thicknesses of asphalt and base layers. This deflection difference tend to be uniquely related to the combined effect of asphalt concrete and base course. However, it was difficult to clearly discriminate the effect of these two layers. For the asphalt concrete layer, there are two ways to predict the modulus. Firstly, the elastic or resilient modulus of asphalt concrete was characterized by its relationship with asphalt constant power viscosity which varies with temperature. It was found, by Ruth et al., that the relationship, the log constant power viscosity versus log temperature (°K), was linear (4). Secondly, the asphalt concrete modulus can be predicted directly from the deflection difference along the transverse direction. Therefore, base course modulus can be solved from asphalt concrete modulus E_1 and composite modulus E_{12}.

In addition, the sensitivity analyses had also demonstrated that the farthest geophone position in the system could be uniquely related to the subgrade modulus. Attempts were therefore made to identify similar unique positions for subbase modulus prediction. The lack of sensitivity of the subbase course with sensor deflections implies the difficulty of developing simple prediction equations.

However, it was found that the deflection difference between geophone positions $D_{x/36}$ and $D_{x/60}$ eliminated the effect of the E_1 and E_2 layers. The difference in the deflections was

primarily dependent upon the moduli of subbase and subgrade and thicknesses of the asphalt concrete and base layers. The subgrade moduli could be predicted by geophone position $D_{x/60}$. The differential deflection between sensors $D_{x/36}$ and $D_{x/60}$, and the deflection at sensor $D_{x/60}$ were probably key parameters related to the subbase. Table 6.1 summarized the results of sensitivity analysis results.

Table 6.1 Sensitivity Analyses Results

Deflection Difference or Deflection	E_1	E_2	E_3	E_4	t_1	t_2	t_3
$D_{y/0}$ - $D_{x/8}$	✓	✓			✓	✓	
$D_{x/60}$				✓			
$D_{x/36}$ - $D_{x/60}$			✓	✓	✓	✓	✓
$D_{y/0}$ - $D_{y/8}$	✓				✓		

6.3.2 Model Formation (**Step 2**)

Having chosen a subset of k independent variables to be candidates for inclusion in the regression analyses and the dependent variable E (moduli), **Step 2** involved refining the information gleaned from **Step 1** and estimation of the empirical relationships between the dependent and independent variables. This step employed the following procedure: (1) draw scatter diagrams, sort out the relationship between the deflections or deflection differences and each of the pavement parameters (layer modulus and thickness); (2) set up fundamental regression models for each of the moduli and specific deflection(s) and/or deflection difference(s),

respectively, at a fixed layer thickness, and (3) develop general regression equations by SAS program (72).

The scatter plots shown in Figures 6.8 through 6.11 illustrate the association between the specific deflection or deflection difference (**Step 1**) and one of the pavement variables (layer modulus and thickness) which might be having some influence. In each of the diagrams, the deflection or deflection difference is on the vertical axis and pavement variable is on the horizontal axis.

A visual inspection of the four diagrams reveals the following:

1. Figure 6.8 indicated that the deflection difference $D_{y/0}$ - $D_{y/08}$ is only related to asphalt concrete layer modulus and thickness. The general relationship can be assumed as a power log function; i.e. $\log(E_1) = B_0 + B_1\log(D_{y/0} - D_{y/08})$. Coefficients B_0 and B_1 are dependent on pavement layer thicknesses t_1. A coefficient of determination R^2 for this function was only 0.80. Therefore, another regression model, $\log(E_1) = B_0 + B_1\log(D_{y/0} - B_2*D_{y/08})$, was used to improve the coefficient of determination that resulted in an R^2 of 0.934.
2. Figure 6.9 indicate that deflection difference $D_{y/0}$ - $D_{x/8}$ is only influenced by composite modulus E_{12}, asphalt concrete layer thickness t_1 . The general relationship of composite modulus E_{12} with deflection difference $D_{y/0}$ - $D_{x/8}$ is power log function ($\log(E_{12})=B_0 + B_1\log(D_{y/0} - D_{x/8})$). Coefficients B_0 and B_1 are dependent on pavement layer thicknesses t_1 and t_2.

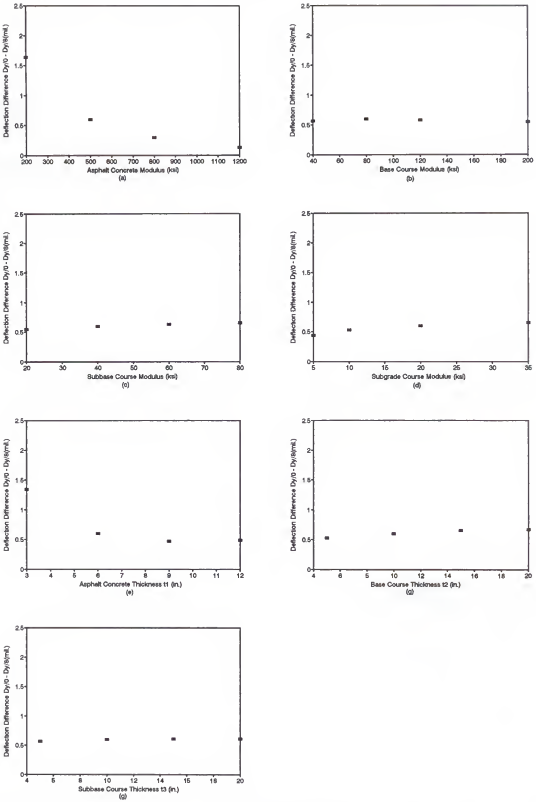


Figure 6.8 Deflection Difference ($D_{y/0}$-$D_{y/8}$) versus Pavement Parameters

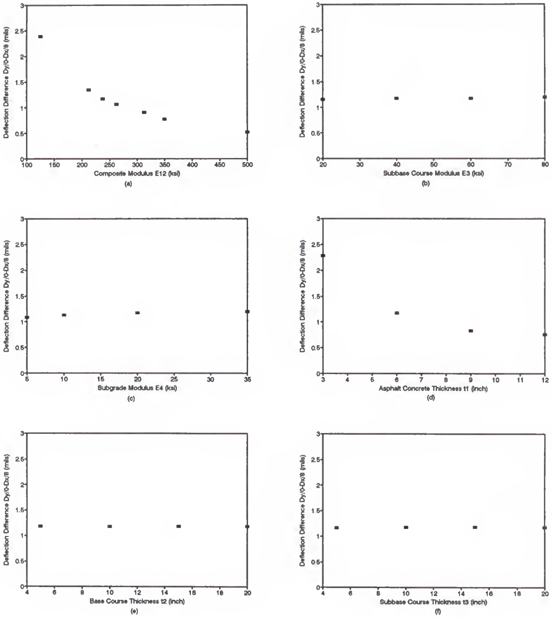


Figure 6.9 Deflection Difference ($D_{y/0}$-$D_{x/8}$) versus Pavement Parameters

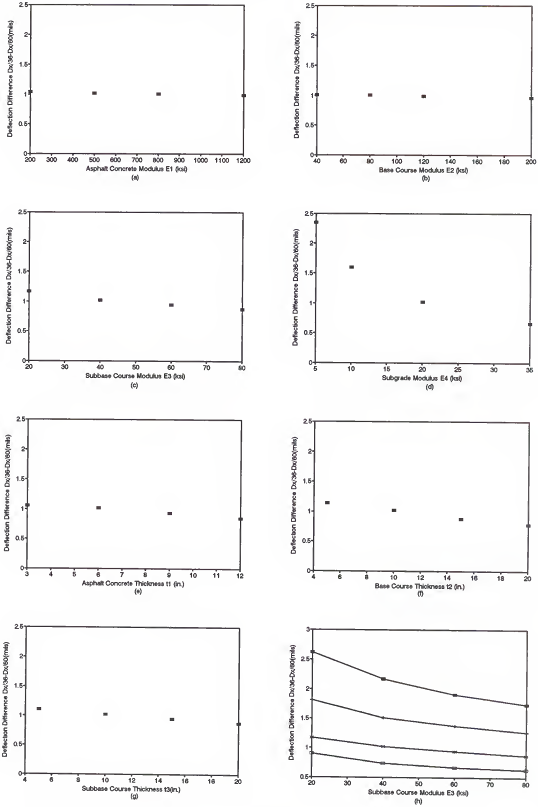


Figure 6.10 Deflection Difference ($D_{x/36}$-$D_{x/60}$) versus Pavement Parameters

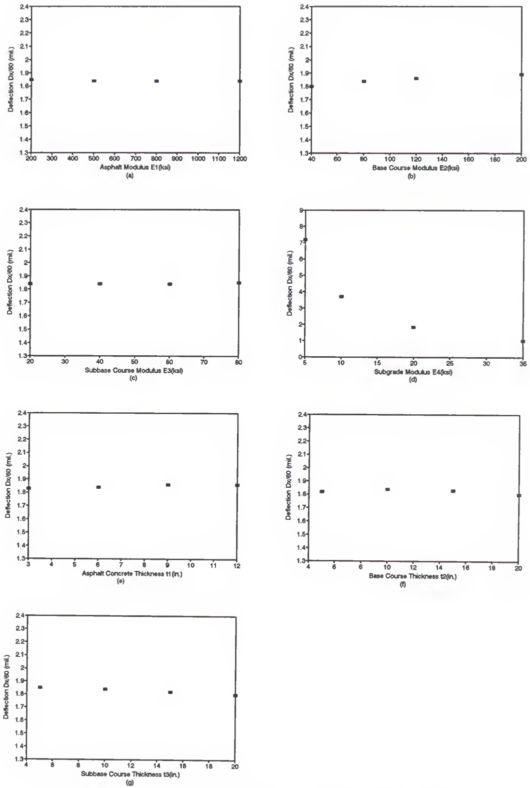


Figure 6.11 Deflection ($D_{x/60}$) versus Pavement Parameters

3. Figure 6.10 indicates that deflection difference $D_{x/36}$ - $D_{x/60}$ is influenced by subbase modulus (E_3), subgrade modulus E_4, asphalt concrete layer thickness t_1, base course layer thickness t_2. The deflection difference is also slightly influenced by base course modulus (E_2) and asphalt concrete modulus(E_1). In addition, the nonparallel lines of Figure 6.10 (h) show that the expected change in $D_{x/36}$ - $D_{x/60}$ for a unit change in E_3 depends on the level of E_4, which are said to interact. The subgrade modulus E_4 is related to $D_{x/60}$, base course modulus E_2 and asphalt concrete modulus E_1 are related to deflection difference $D_{y/0}$ - $D_{x/12}$, therefore, the general regression model is assumed to be $\log(E_3) = B_0 + B_1\log(D_{y/0} - D_{x/12}) + B_2\log(D_{x/36} - D_{x/60})$. Coefficients B_0, B_1 and B_2 are dependent of pavement layer thicknesses t_1 and t_2 and deflection $D_{x/60}$. SAS regression analysis indicated that this regression model had a low coefficient of determination (R^2 = 0.88). Therefore, a new regression model, $\log(E_3) = B_0 + B_1\log(D_{y/0} - B_2*D_{x/24}) + B_3\log(D_{x/36} - D_{x/60})$, was used in attempting to improve the correlation. The improved coefficient of determination (R^2) was 0.94.

4. Figure 6.11 indicates that deflection $D_{x/60}$ is only influenced by subgrade modulus E_4. The relationship between E_4 and $D_{x/60}$ is also a power log function $\log(E_4) = B_0 + B_1\log(D_{x/60})$. B_0 and B_1 are constant in this particular situation.

The SAS program was used to develop the tentative modulus prediction equations based on these analyses of the dual-load FWD with 24-in spacing. SAS application programming and nonlinear regression analysis results are illustrated in Appendix A. The summary results of regression analysis are described below:

FWD Dual Load (8,000 lbs total)

1. Asphalt concrete layer moduli E_1:

$$E_1 = 2246.7047(t_1)^{(-0.9911)}(D_{y/0} - 0.975D_{y/08})^{(-0.7788-1.33807/t1)} \quad \text{Eqn. 6.1}$$

$R^2 = 0.934$

2. Layer moduli E_{12} (composite of E1 and E2):

$$E_{12} = 2285.80(t_1)^{-0.6417}(t_2)^{-0.3722}(D_{y/0} - 0.99D_{x/8})^{-1.2862+0.053t1} \quad \text{Eqn. 6.2}$$

$R^2 = 0.961$

where $E_{12} = (E_1 t_1 + E_2 t_2)/(t_1 + t_2)$

3. Subgrade Moduli E_4:

$$E_4 = 45.90 \ (D_{x/48})^{-1.05} \quad R^2 = 0.995 \quad \text{Eqn. 6.3}$$

$$E_4 = 36.04 \ (D_{x/60})^{-1.00} \quad R^2 = 0.998 \quad \text{Eqn. 6.4}$$

4. Subbase Moduli E_3:

$$E_3 = 85.1656(t_2)^{-1.0889}(D_{y/0} - 1.1D_{x/24})^{1.2151 - 0.7249/(Dx/60) - 4.9186/t2}$$
$$* (D_{x/36} - D_{x/60})^{-4.0581}(D_{x/60})^{1.764 + 0.8077/t1 + 3.9683/t2} \quad \text{Eqn. 6.5}$$

$R^2 = 0.9413$

These results clearly indicated that a dual loading system is very useful for discriminating between the near-surface layer moduli. This demonstrated that the FWD dual system with 24-inch load spacing is better than a single loading. However, further research was needing to develop pavement layer modulus

prediction equations of a FWD system with a wider load spacing for comparison.

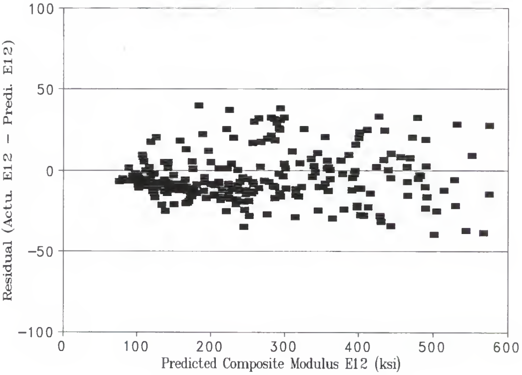
6.3.3 Residual Analysis: Checking Regression Analysis (**Step 3**)

The final step in the process was to check the validity of the regression models. Graphics were used to complete this step. First, scatterplot of E_i versus independent (deflection and/or deflection difference, pavement thickness) was used to see if a higher-order model would be a more appropriate model for the relationship between E_i and independent (**Step 2**). Second, a plot of residuals E_i - E_i' versus predicted values E_i' may give an indication of the following problems:

1. Outliers. A visual examination of the residual plot will identify data points with unusually high (in absolute value) residuals.

2. Violation of the assumptions of the fitted model. The regression model is assumed as a relationship between y and the independent variables x_i with independent, normally distributed errors at a constant variance.

The residual plot for the regression models in **Step 2** and data set that has none of these apparent problems would look much like the plots in Figures 6.12 through 6.14. In other word, there are no extremely large residuals (and hence no apparent outliers) and that there are no trend in the residual to indicate that the models are inappropriate. However the plot of residuals against the predicted values in Figure 6.15 shows a distinct pattern as the residuals moves from positive to



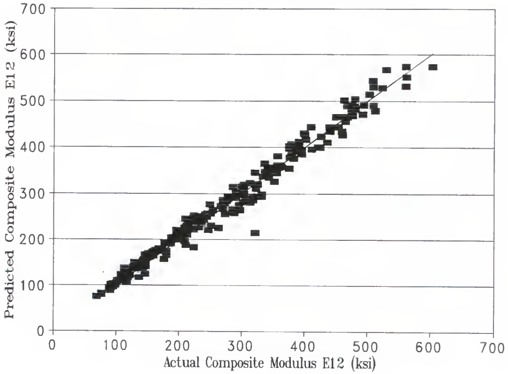
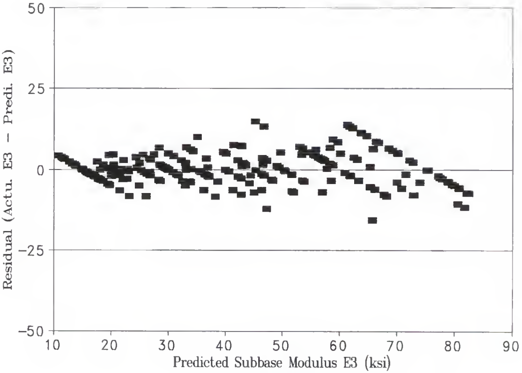


Figure 6.12 Residuals versus Predicted E_{12} Values



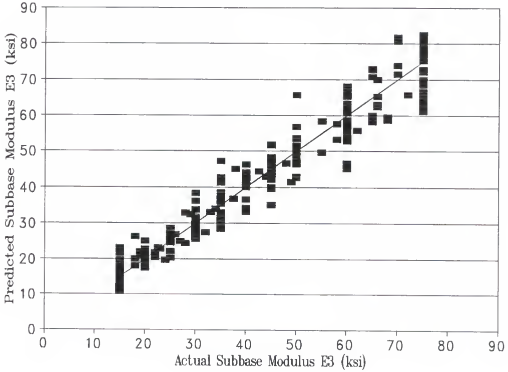
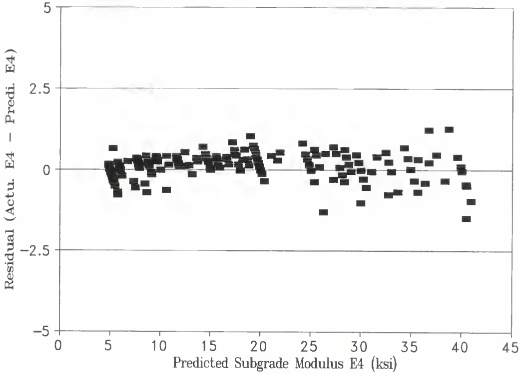


Figure 6.13 Residuals versus Predicted E_3 Values



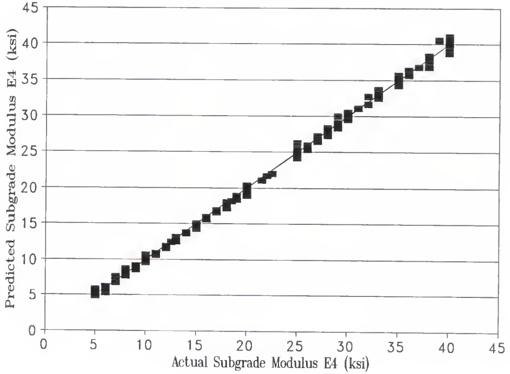


Figure 6.14 Residuals versus Predicted E_4 Values

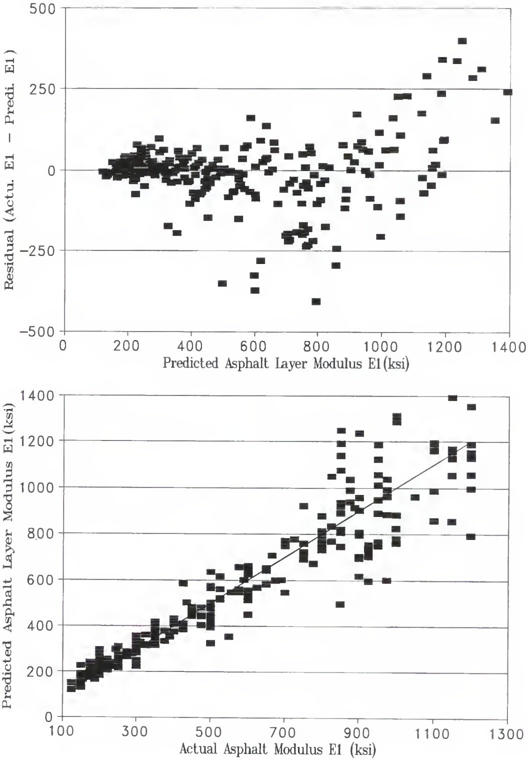


Figure 6.15 Residuals versus Predicted E_1 Values

negative to positive again. This structure suggests that additional terms may be needed in the asphalt concrete modulus prediction regression model. Since the interaction between asphalt concrete and base course modulus may have an effect on the deflection difference $D_{y/0}$ - $D_{y/08}$, the deflection difference $D_{y/0}$ - $D_{x/8}$ was added to the E_1 prediction model. The new developed asphalt concrete modulus E_1 prediction equation is:

$$E_1 = 442.6067*(D_{y/0} - 0.97D_{y/08})^{(1.5893 - 13.2826/t1)} *(D_{y/0} - D_{x/8})^{(-2.4534+12.2336/t1)} \qquad \text{Eqn. 6.6}$$

$$R^2 = 0.972$$

Figure 6.16 shows a plot of the residuals versus the predicted E_1 values using this new regression model. No unusual structure is apparent. Therefore, this new regression model is appropriate.

6.4 The Predictions of the Base Course Modulus E_2

Sensitivity analyses indicated that the base course modulus was related to the deflection basin in the vicinity of the loads in the longitudinal direction. However, there was a definite influence of the asphalt concrete modulus in the relationship between deflections and base course modulus, as illustrated in Figures 6.1 and 6.2. Therefore, there was no analytical solution that would provide layer modulus E_2 directly from the measured deflections. Base course modulus

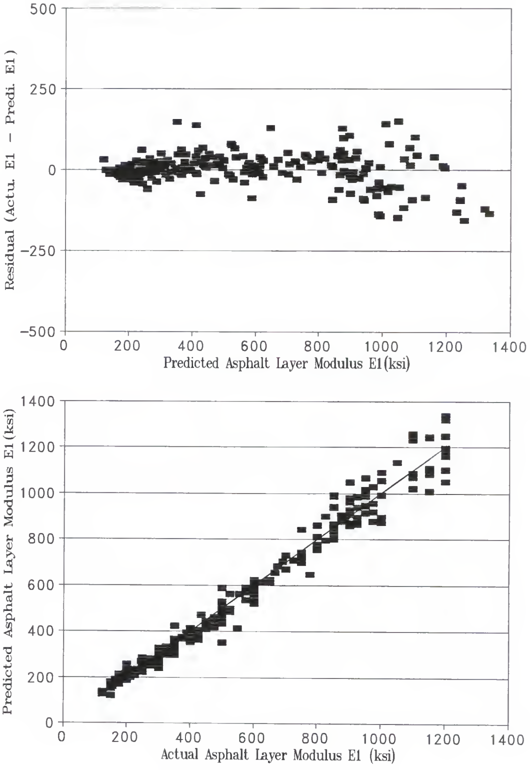


Figure 6.16 Residuals versus Predicted E_1 Values (Eqn. 6.6)

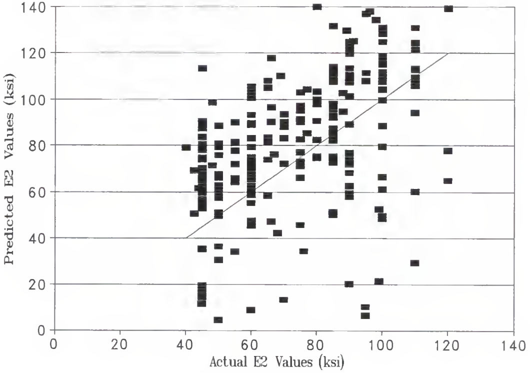


Figure 6.17 E_2 Predictions from the Weighting Equations

E_2 must be solved from the weighting equation, composite modulus E_{12} and asphalt concrete modulus E_1. Figure 6.17 shows the results of layer modulus E_2 predictions. In this figure, E_2 values were predicted from the weighting equation using predicted values of E_1 and E_{12} as input.

Overall assessment of the E_2 predictions indicated that some E_2 values predicted from the weighting equation are much different than the actual values. This suggests that the E_2 values obtained from the weighting equation may result in substantial prediction errors. This discrepancy was suspected

to be due to the inadequacy of the approximate formula used to calculate the composite modulus E_{12}. To support this contention, the Barro's (73) approximate formula for composite modulus was then used to calculate the composite modulus. The new composite modulus (E_{12}) regression equation based upon Barro's weighting equation was developed as the follows:

$$E_{12} = (D_{y/0} - D_{x/12})^{2.5058-7.0175/t1-15.3592/t2}$$

$$\times\ (D_{y/0} - 1.03D_{x/20})^{-3.3778+6.5149/t1+15.4336/t2} \qquad \text{Eqn. 6.7}$$

$$R^2 = 0.97$$

where $E_{12} = [(E_1^{1/3}t_1 + E_2^{1/3}t_2) / (t_1 + t_2)]^3$ (Barro's Weighting Equation)

The above weighting equation, proposed to calculate the composite modulus E_{12}, can be substituted for moduli E_1 and E_2 for the same deflection.

Figure 6.18 illustrates the comparison of predicted E_2 values and the actual E_2 values. This comparison indicated that the prediction errors were reduced substantially as compared to the results shown in Figure 6.17. The prediction residuals for E_2 are, in general, within 20 ksi. However, some large errors do stand out, especially when the E_2 values are low. Therefore, the best way to obtain the base course modulus E_2 is to use the predicted values as seed moduli in a back calculation procedure. In a four layer system, the iteratively obtained solution for the modulus of the base course will be unique if the moduli of the asphalt concrete, subbase course and subgrade are used as predicted.

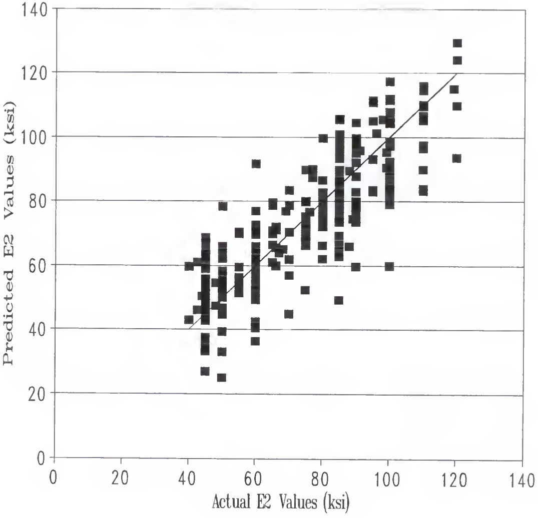


Figure 6.18 Comparison Between Actual and Predicted Base Course Moduli

CHAPTER 7
PREDICTION EQUATIONS FOR FWD WITH 40-INCH LOAD SPACING

7.1 Introduction

Prior research conducted by Roque etc. (5) indicated that a dual-load-rigid-plate system, spaced at 40 inches, could improve the separation of near-surface layer effects as compared to the other FWD configuration systems. The dual load system should provide superior layer discrimination. This chapter will focus on the development of a set of tentative equations to estimate pavement layer moduli from surface deflections produced by the FWD system. Additionally, an attempt will be made to elaborate on how this system works.

The research approach in this chapter will follow the same steps used in Chapter 6. Firstly, the FWD with a 40-inch load spacing and different geophone locations was simulated in the RIGID plate approximation program to predict deflection values for four-layer pavement systems. Secondly, a sensitivity analysis was performed to determine a superior FWD configuration system with the 40 load spacing. This analysis also provided information for finding the key parameters for the development of tentative prediction equations. Finally, the RIGID program was used to simulate the FWD configuration system so as to develop a broad number of deflection databases

with different pavement parameter combinations. The computer program SAS was used to develop layer modulus prediction equations.

In this chapter, the deflections predicted by the program were referred to as the "actual deflections", and the moduli used as input were referred to as the " actual moduli". These "actual" variables were used to develop a set of regression equations. Thus, the moduli determined by the prediction equations were the "predicted moduli", and deflection obtained from the computer program using the "predicted moduli" were the "predicted deflections". The goodness of fit for the regression models was determined by comparing the actual moduli with the predicted values.

7.2 Sensitivity Analysis of Theoretical FWD Deflection Basins

7.2.1 Parametric Study

Analyses were conducted to determine the sensitivity of the dual-load system with 40-inch load spacing. A four layer pavement system including seven parameters (four layer moduli, three layer thicknesses) was used in this analyses. In this study, each layer modulus or thickness was varied while the others were kept constant. For example, the modulus of the asphalt concrete changed from 200 ksi to 1200 ksi without changing the base, subbase, and subgrade moduli and the layer thicknesses. The load used in the sensitivity study was 8,000 pounds. The RIGID program was then used to calculate the deflection basins. The basins gave a good indication of how

sensitive the system was and where the sensors should be positioned to achieve superior layer discrimination.

7.2.2 Variations in Pavement Parameters

Figure 7.1 shows the effect of change in asphalt concrete modulus on the deflection basins. It can be seen that asphalt concrete layer modulus had an effect on the shape of the transverse deflection basin. However, almost no apparent effect was observed in the longitudinal direction beyond the 20 inch offset distance. A change in layer modulus was mainly reflected in the deflections under the loading center. Similar analyses for other layer moduli and thicknesses clearly indicated that only asphalt concrete layer thickness had sizeable effect on the shape of the transverse deflection basin. Therefore, it was apparent that the shape of the transverse deflection basin could be related to the asphalt concrete modulus.

Figure 7.2 shows the effect of change in base course modulus on the deflection basins. It indicated that base course modulus had very limited effect on the shape of the transverse deflection basin. However, It had apparent effect on the longitudinal deflections closer to the loads. The most considerable change occurred at a zero offset distance along the longitudinal direction. The effect was not apparent beyond 30 inch offset distance along the longitudinal direction. Further analysis for other layer moduli and thicknesses indicated that asphalt concrete layer modulus and asphalt layer and base course layer thickness (t_1 and t_2)

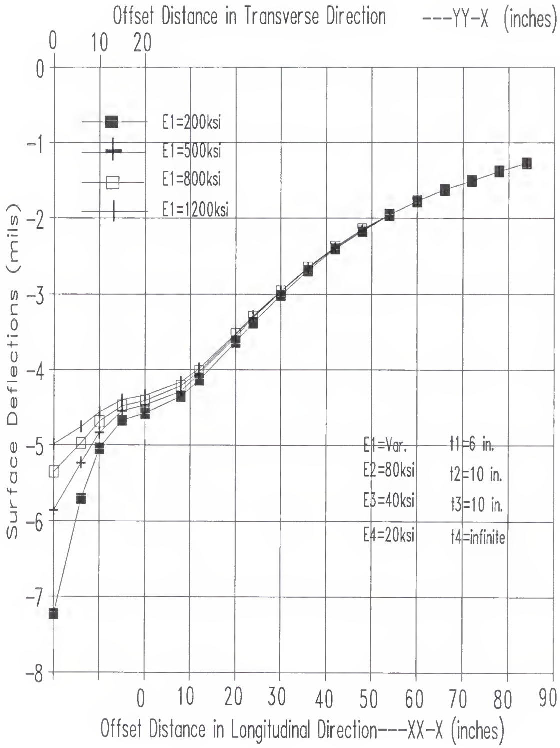


Figure 7.1 Effect of Variable Asphalt Concrete Layer Moduli on Surface Deflections with 40-inch Load Spacing

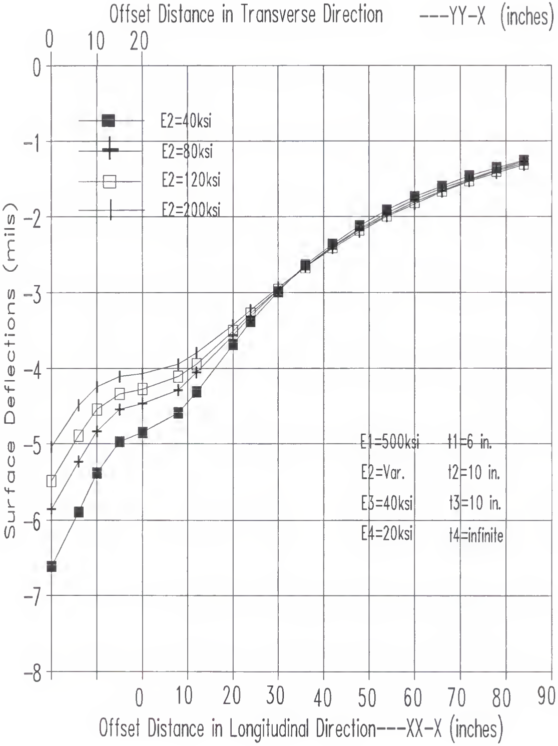


Figure 7.2 Effect of Variable Base Course Moduli on Surface Deflections with 40-inch Load Spacing

also had an effect on the shape of deflection basins close to the loads of the longitudinal direction. This appeared to indicate that the base modulus was not the only parameter relating to that portion of the longitudinal deflection basin close to the applied loads. It would be difficult to reliably determine the base course moduli on the basis of measured deflection deflections from the system. Therefore, a combined modulus for E_1 and E_2 (composite modulus E_{12}) was used to establish a relationship with deflection difference between $D_{y/0}$ and $D_{x/12}$ or $D_{y/0}$ and $D_{x/8}$. Base course modulus E_2 can be solved with the weighting equations E_1 and E_{12}.

Figure 7.3 shows that subbase course modulus has no effect on the shape of the transverse deflection basins. A change in the subbase modulus was reflected only in the magnitude of these deflections. In the longitudinal direction, differences in deflections were observed only up to about 48 inch along the centerline. Beyond this distance, the deflections were almost identical. This implied that the subbase could be related to the portion of the longitudinal deflection basin further away from the loads than the base course. In contrast to what was observed in the other Figures, discrimination of the effect of subbase course modulus was not clear.

The changes in the subgrade moduli, as shown in Figure 7.4, indicated that the effect of the subgrade modulus was primarily on the magnitude of all deflections. The shape of

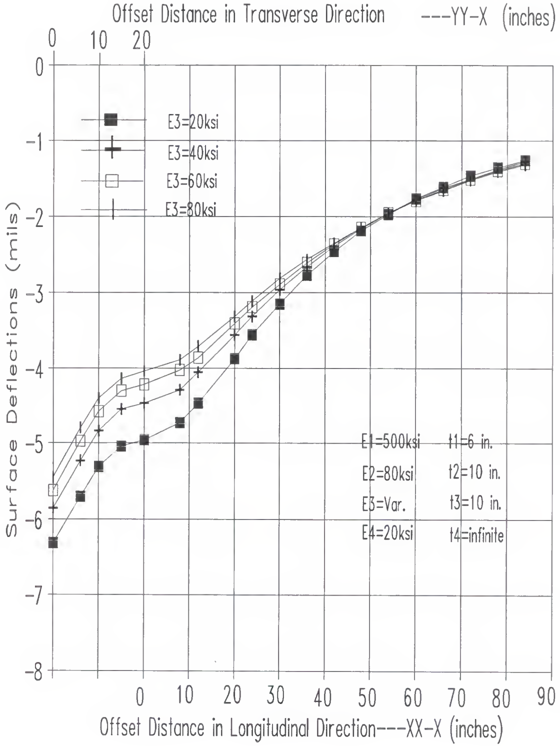


Figure 7.3 Effect of Variable Subbase Course on Surface Deflections with 40-inch Load Spacing

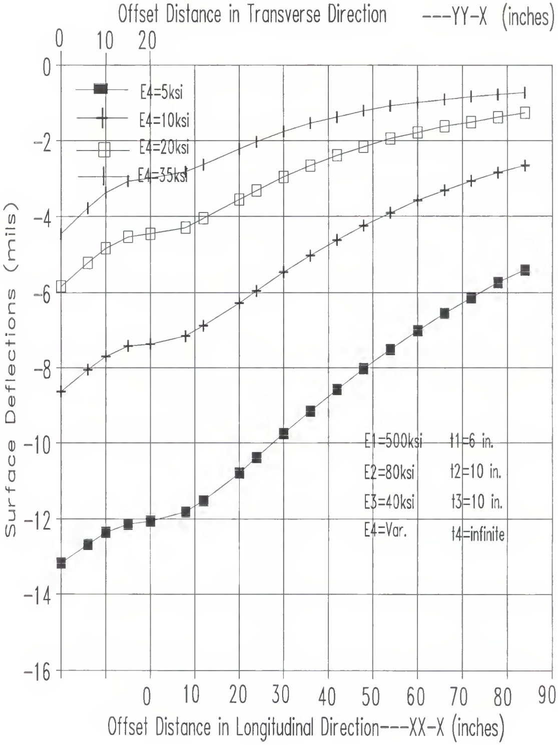


Figure 7.4 Effect of Variable Subgrade Moduli on Surface Deflections with 40-inch Load Spacing

the transverse deflection basin was the same for the different subgrade modulus. In the longitudinal direction, the subgrade modulus still had considerable effect on the deflection at a distance of over 60 inches, while other layer moduli had no discernable effect at this distance. Therefore, the deflections at distances of 60 and 72 inches could be uniquely related to the subgrade modulus.

Similarly, a sensitivity study was also performed to assess the effects of layer thicknesses on the deflection basins. Figures 7.5 through 7.7 show the results of the analyses. According to these Figures, asphalt, base course, and subbase course layer thicknesses had an effect on the transverse deflections but almost no effect longitudinally beyond 48 inches. Furthermore, it seemed that base course and subbase course thicknesses did not have an effect on the difference between $D_{y/0}$ and $D_{x/12}$ as illustrated in Figures 7.6 and 7.7. This somewhat unique condition indicates that the deflection response ($D_{y/0}$ - $D_{x/12}$) is dependent only upon E_1, E_2 and t_1. The reason for this probably lies in the fact that $D_{x/12}$ is at or close to the inflection point of the deflection basins. Further research indicated that when t_1 is reduced to 3 in. and the combined thicknesses of t_2 and t_3 are 20 in. or less, there is a shift in deflection magnitude with t_2. This implies that some location other than $D_{x/12}$ is the inflection point.

These sensitivity analyses indicated that specific deflections or deflection differentials from the dual-load

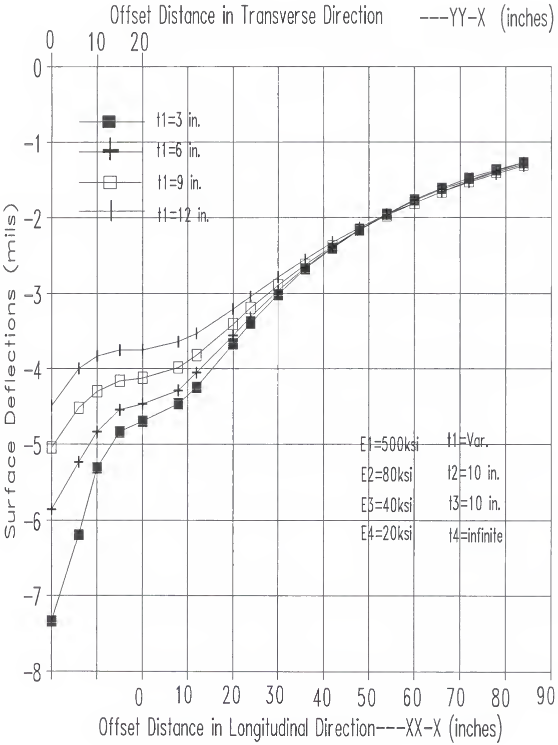


Figure 7.5 Effect of Variable Asphalt Concrete Layer Thickness on Surface Deflections with 40-inch Load Spacing

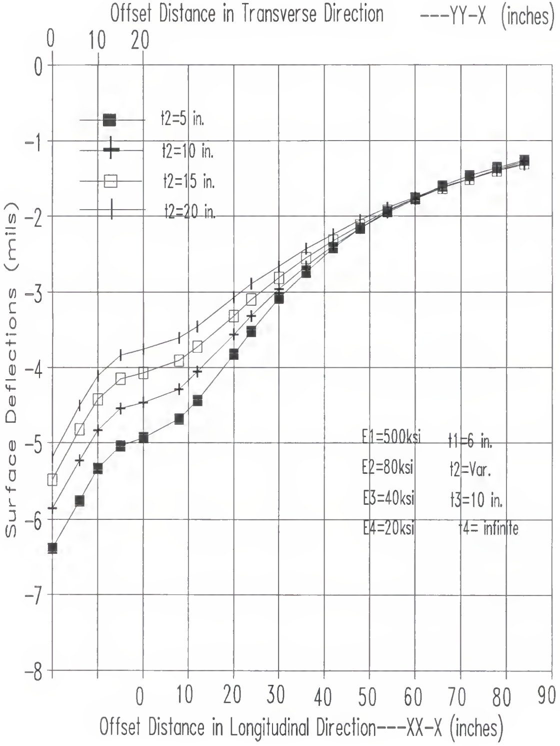


Figure 7.6 Effect of Variable Base Course Thickness on Surface Deflections with 40-inch Load Spacing

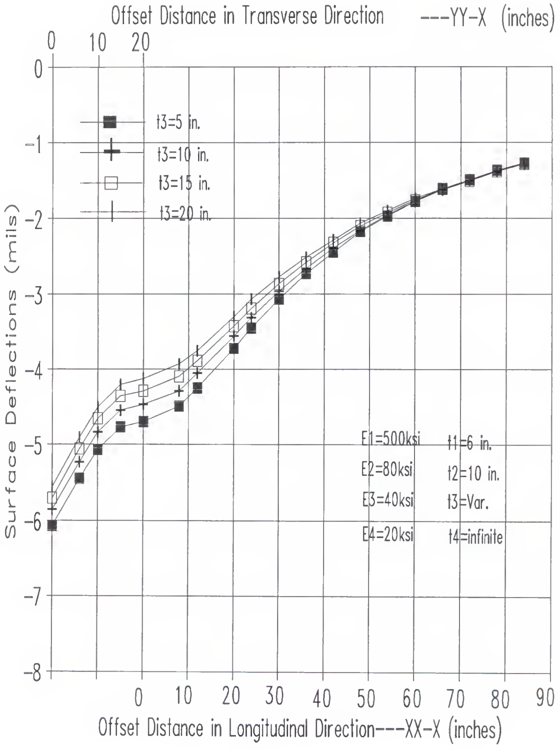


Figure 7.7 Effect of Variable Subbase Course Thickness on Surface Deflections with 40-inch Load Spacing

system could be related to the stiffness (modulus and thickness) of specific pavement layers. Based on this study, a set of comprehensive equations were developed to predict layer modulus for the range of parameters given in Table 5.2.

7.3 Development of Layer Moduli Prediction Equations

The elastic-layer computer program RIGID was used to generate FWD deflection basins for the 40-in dual-load-rigid-plate configuration using different combination of moduli and thicknesses covering the range listed in Table 5.2. These deflection data (about 14,300 deflection basins) were then used to develop regression equations based on sensitivity analysis.

The development of layer moduli prediction equations were done with three steps: (1) Sort out candidate independent variables; (2) Select the form for a regression model involving some of these variables; and (3) check the violation of certain regression assumptions.

7.3.1 Selecting the Variables (**Step 1**)

Step 1 was completed by performing parametric sensitivity analyses as described earlier. The sensitivity study indicated that the difference between sensor $D_{y/0}$ and $D_{y/10}$ deflection (i.e. $D_{y/0}$ - $D_{y/10}$) essentially eliminated the effect of the underlying layers and was primarily dependent upon the moduli and thicknesses of asphalt and base layers. This deflection difference tends to be uniquely related to the effect of asphalt concrete.

In addition, the sensitivity analyses had also demonstrated that the farthest geophone positions ($D_{x/60}$, $D_{x/72}$) in the system could be uniquely related to the subgrade modulus. Therefore, attempts were made to identify similar unique positions for base and subbase modulus prediction. For the base course modulus, it was relatively easy to develop the composite modulus prediction equation using combined asphalt concrete and base course modulus with the deflection difference $D_{y/0}$ - $D_{x/12}$; then the base course modulus can be solved by the weighting equations, composite modulus and asphalt concrete modulus. However, the lack of sensitivity of the subbase course with sensor deflections produced extreme difficulty in developing simple prediction equations. Table 7.1 summarized the sensitivity analysis results.

Table 7.1 Sensitivity Analyses Results

Deflection Difference or Deflection	E_1	E_2	E_3	E_4	t_1	t_2	t_3
$D_{y/0}$ - $D_{y/10}$	✓	✓			✓		
$D_{y/0}$ - $D_{x/12}$	✓	✓		✓	✓		
$D_{x/60}$, $D_{x/72}$				✓			
$D_{x/36}$ - $D_{x/60}$			✓	✓		✓	✓

7.3.2 Model Formation (**Step 2**)

This section will focus on refining the information gleaned from **STEP 1** for the development of actual relationship between the dependent and independent variables.

The scatter plots 7.8 through 7.12 illustrate the association between the specific deflection or deflection difference (**Step 1**) and those pavement variables (layer modulus or thickness) which might have some influence on deflection response. In each of the diagrams, the deflection or deflection difference is on the vertical axis and pavement variable is on the horizontal axis.

A visual inspection of the five diagrams reveals that:

1. Figure 7.8 shows that deflection difference $D_{y/0} - D_{y/10}$ is influenced by asphalt concrete modulus E_1, asphalt concrete layer thickness t_1 and base course modulus E_2. Low subgrade modulus values also had effect on the deflection difference. Therefore, the relations for asphalt concrete layer moduli were not as straight forward as the subgrade and composite modulus relations. Several attempts were made to find an equation that related the surface deflections to the asphalt concrete modulus. The best results were obtained using the deflection differences $D_{y/0} - D_{y/10}$ and $D_{y/0} - D_{x/12}$ /or $D_{y/0} - D_{y/20}$ at certain pavement thickness. The general E_1 prediction equations are: (1) $t_1 + 0.2t_2 \le 7$in.

$\log(E_1) = B_0 + B_1\log(D_{y/0} - B_2{*}D_{y/10}) + B_3\log(D_{y/0} - B_4{*}D_{y/20}) + B_5\log(D_{y/10} - B_6{*}D_{y/20})$

(2) $t_1 + 0.2t_2 > 7$in.

$\log(E_1) = B_0 + B_1\log(D_{y/0} - B_2{*}D_{y/10}) + B_3\log(D_{y/0} - B_4{*}D_{y/12})$

Serials B are the functions of pavement thicknesses.

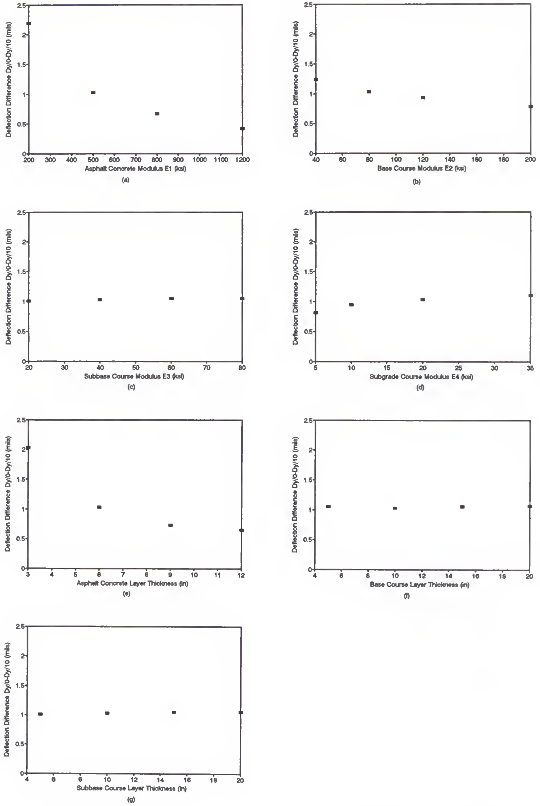


Figure 7.8 Deflection Difference ($D_{y/0}$-$D_{y/10}$) versus Pavement Parameters

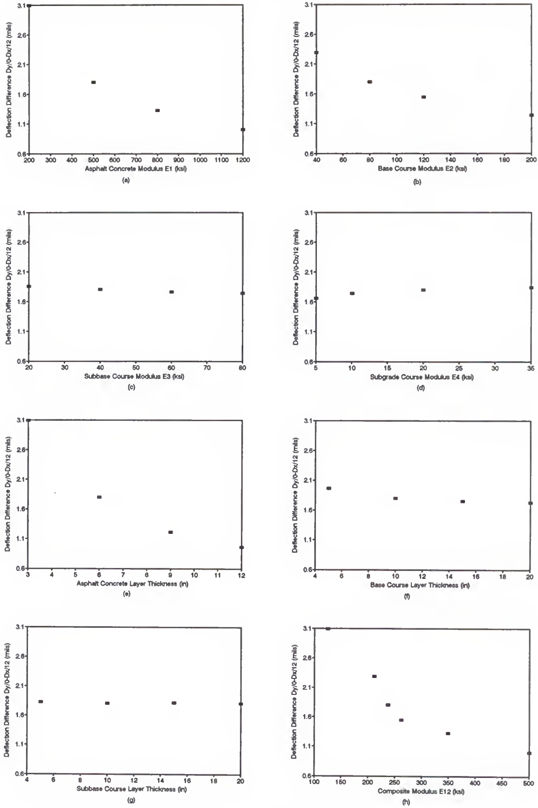


Figure 7.9 Deflection Difference ($D_{y/0}$-$D_{x/12}$) versus Pavement Parameters

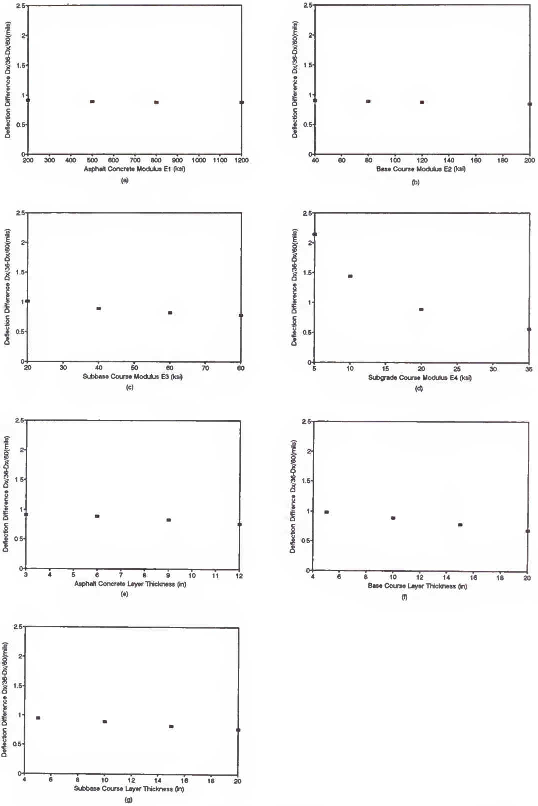


Figure 7.10 Deflection Difference ($D_{x/36}$-$D_{x/60}$) versus Pavement Parameters

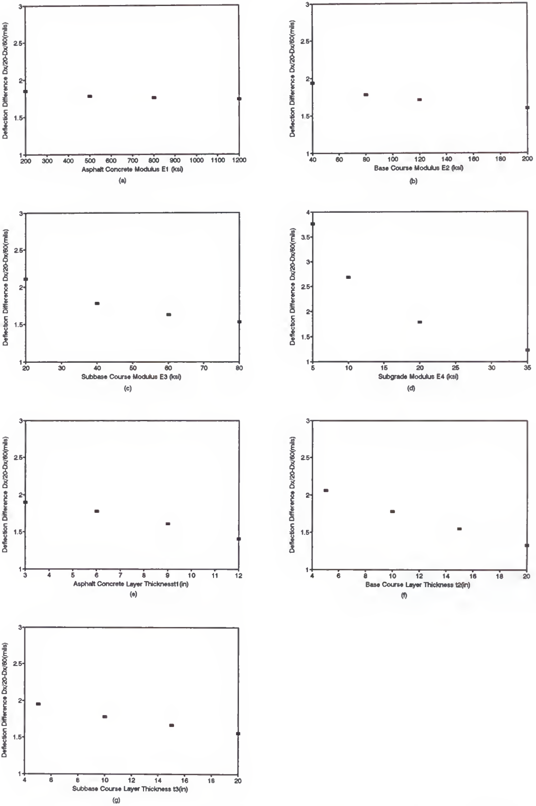


Figure 7.11 Deflection Difference ($D_{x/20}$-$D_{x/60}$) versus Pavement Parameters

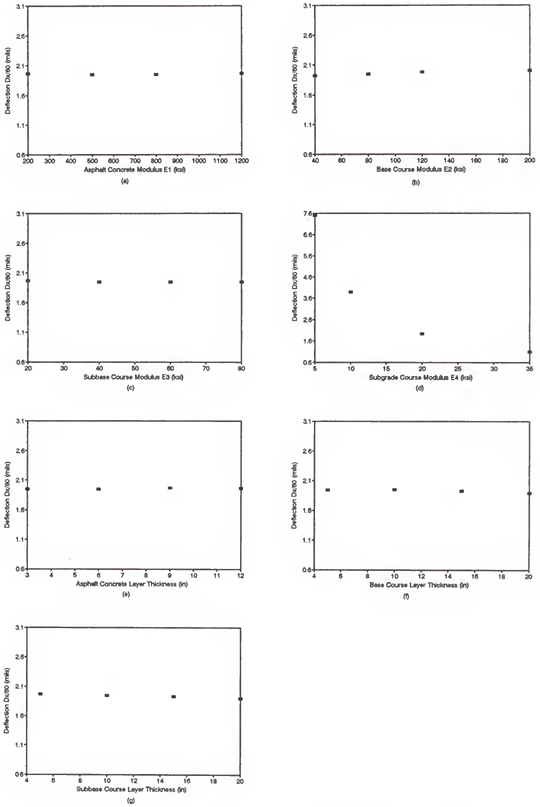


Figure 7.12 Deflection Difference ($D_{x/60}$) versus Pavement Parameters

2. Figure 7.9 indicates that deflection difference $D_{y/0}$ - $D_{x/12}$ is only influenced by composite modulus E_{12}, asphalt concrete layer thickness t_1 and base course layer thickness t_2. The general relationship of composite modulus E_{12} with deflection difference $D_{y/0}$ - $D_{x/12}$ is power log function $\log(E_{12})=B_0 + B_1\log(D_{y/0} - B_2*D_{x/12})$. Coefficients B_0 and B_1 are dependent on pavement layer thicknesses t_1 and t_2. This model provided good predictions in most cases, but did a poor job for certain cases when the asphalt pavement is thin (t_1+0.2t_2 < 7in.) With the exception of the case cited, the coefficient of determination (R^2) was 0.98. For the thin pavement, the model was $\log(E_{12})=B_0 + B_1\log(D_{y/0} - B_2*D_{x/12}) + B_3\log(D_{y/0} - B_4D_{y/10})$ with a coefficient of determination of 0.97.

3. Figure 7.10 indicates that deflection difference $D_{x/36}$ - $D_{x/60}$ is influenced by subgrade modulus E_4. The deflection difference is also slightly influenced by base course modulus (E_2), subbase modulus (E_3), and pavement thickness. The effects of the subbase could not be distinguished. Therefore, further effort was devoted to using other deflection difference such as $D_{x/20}$ and $D_{x/60}$ (Figure 7.11). The effects of the subbase course modulus were still not clear. This indicated that large load spacing weakened the effect of the subbase course modulus on the surface deflections.

4. Figure 7.12 indicates that deflections $D_{x/60}$ and $D_{x/72}$ are only influenced by subgrade modulus E_4. The relationship between E_4 and $D_{x/60}$/or $D_{x/72}$ is also a power log function $\log(E_4)$

$= B_0 + B_1 \log(D_{x/60})$ or $\log(E_4) = B_0 + B_1 \log(D_{x/72})$. B_0 and B_1 are constant in this particular situation.

Based on these analyses of the dual-load FWD with 40-in spacing, the SAS program was used to develop the tentative modulus prediction equations. SAS application programming and nonlinear regression analysis results are illustrated in Appendix B. The summary results of regression analysis are described below:

FWD Dual Load (8,000 lbs total)

1. Asphalt concrete layer moduli E_1:

(a) $t_1+0.2t_2 \leq 7$in.

$$E_1 = 3768.695 t_1^{-1.111} t_2^{0.313} (D_{y/0}-0.96*D_{y/10})^{(1.287-16.083/t1-2.9803/t2)}$$
$$(D_{y/10} - 0.96*D_{y/20})^{3.287/t1}$$
$$(D_{y/0} - 0.99*D_{y/20})^{(-2.2832+12.649/t1+3.0662/t2)} \qquad \text{Eqn. 7.1}$$

$R^2 = 0.953$

(b) $t_1+0.2t_2 > 7$in.

$$E_1 = 424.256 (D_{y/0}-0.97*D_{y/10})^{(0.4952-15.576/t1+2.3729/t2)}$$
$$(D_{y/0} - D_{x/12})^{(-1.4721+14.6901/t1-1.9733/t2)} \qquad \text{Eqn. 7.2}$$

$R^2 = 0.980$

2. Layer moduli E_{12} (composite of E1 and E2):

(a) $t_1+0.2t_2 \leq 7$in.

$$E_{12} = 198.9477 (t_1)^{0.2393} (D_{y/0} - 0.97D_{y/20})^{-4.1874+13.9511/t1+12.7274/t2}$$
$$(D_{y/0} - 0.97D_{y/10})^{(3.3318-14.3126/t1-12.8208/t2} \qquad \text{Eqn. 7.3}$$

$R^2 = 0.968$

(b) $t_1+0.2t_2 > 7$in.

$$E_{12} = 3543.545 (t_1+0.6348t_2)^{-0.8127} (D_{y/0} - 0.982D_{x/12})^{-0.8638-0.7016/t1}$$

$R^2 = 0.970$ Eqn. 7.4

where: $E_{12} = (E_1 t_1 + E_2 t_2)/(t_1 + t_2)$

3. Subgrade Moduli E_4:

$$E_4 = 35.01\ (D_{x/60})^{-1.01} \quad R^2 = 0.997 \qquad \text{Eqn. 7.5}$$

$$E_4 = 29.72\ (D_{x/72})^{-0.995} \quad R^2 = 0.998 \qquad \text{Eqn. 7.6}$$

These results indicated that this dual loading system is very useful for discriminating between the near-surface layer moduli. Several attempts were made to find an equation that related the surface deflections to the subbase course modulus. However, the FWD system with 40 inches load spacing reduced the effect of subbase course modulus on the surface deflection, which led to having a apparent interacting influence with other layer moduli. No straight forward prediction equation was obtained in this investigation.

7.3.3 Residual Analysis: Checking Regression Analysis (**Step 3**)

Before the conclusions from the analysis of variance can be adopted, the adequacy of the underlying model should be checked. As described before, the primary diagnostic tool is residual analysis. The residuals are

$$e_i = E_i - \hat{E_i}$$

E_i is the actual value and $\hat{E_i}$ is the predicted value.

Figures 7.13 to 7.15 plots the residuals versus the fitted layer modulus values. These plot reveals nothing of unusual interest. There are neither indication of inequality of variance nor evidence pointing to possible outliers. Although the largest residual does stand out somewhat from the others, the problem, however, is not severe enough to have a dramatic impact on the analysis and conclusions.

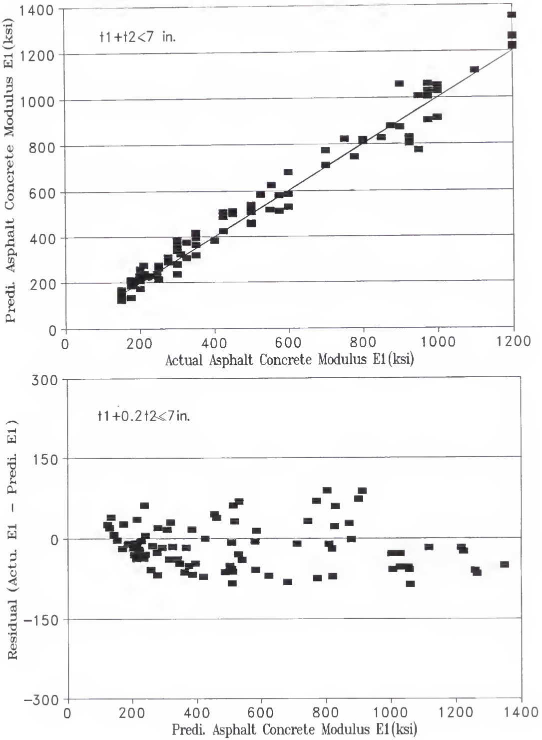
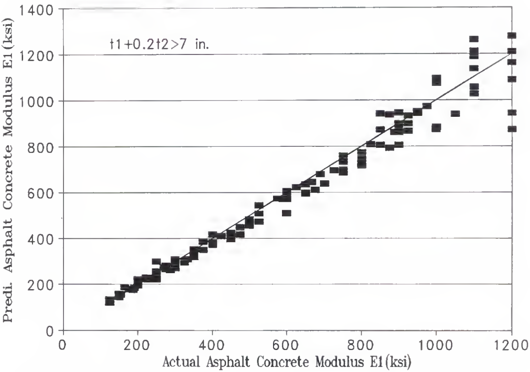


Figure 7.13 Residuals versus Predicted E_1 Values



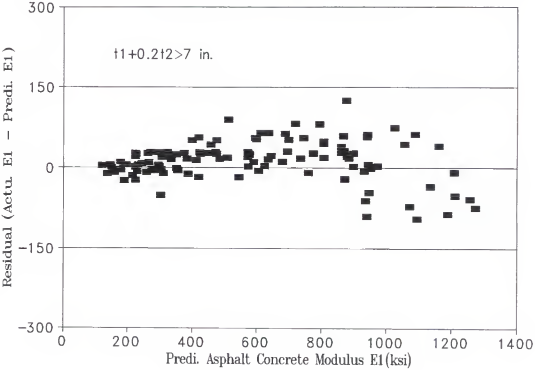


Figure 7.13--Continued

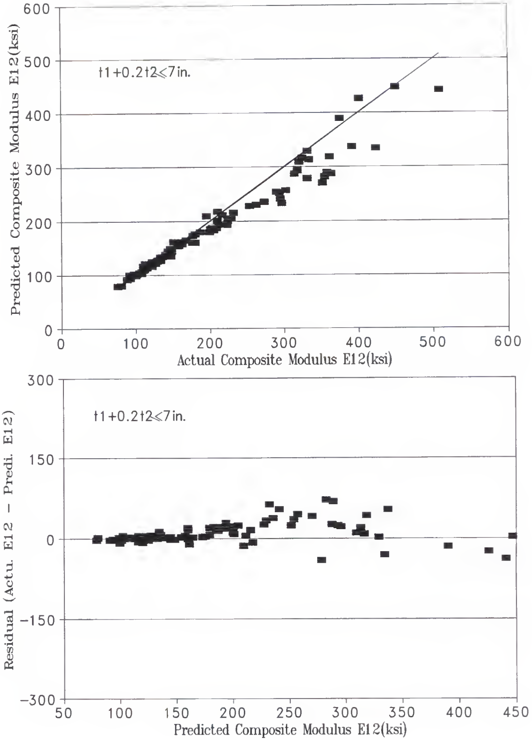
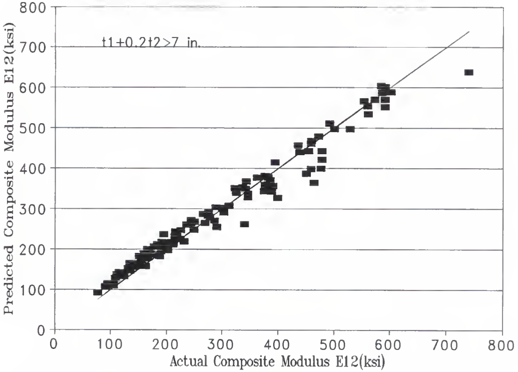


Figure 7.14(a) Residuals versus Predicted E_{12} Values



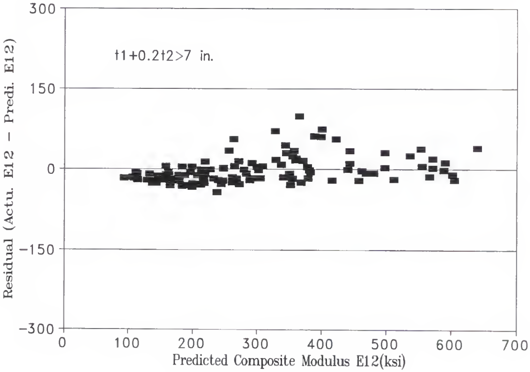


Figure 7.14(b) Residuals versus Predicted E_{12} Values

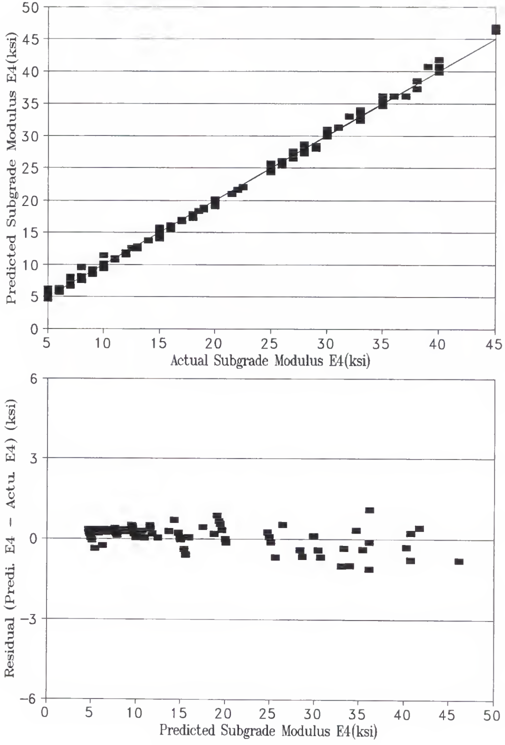


Figure 7.15 Residuals versus Predicted E_4 Values

CHAPTER 8
COMPARATIVE ANALYSIS OF DIFFERENT FWD DUAL-LOAD SYSTEMS

8.1 Introduction

The development of a set of equations to estimate pavement layer moduli from surface deflections produced by dual-load-rigid-plate FWDs with two load spacings has been presented in Chapters 6 and 7. This Chapter deals with the comparison of different dual-load-rigid-plate FWD systems. It will be demonstrated that the effect of load spacing on the layer discrimination facilitates the determination of layer moduli. In addition, the development of a simulation of dual-load system using standard FWD equipment for a direct evaluation of the system is also a part of the study.

8.2 Comparison of FWD Dual-Load Configurations

Previous studies have indicated that a dual-load-rigid-plate system could improve the separation of near-surface layer effects as compared to the single-load FWD. However, differences in discrimination and ability to predict layer moduli were observed with the two dual-load-rigid-plate FWD systems with 24-inch and 40-inch load spacing. These difference were attributed to the spacing between the dual loads. In this section a comparison is made between the

deflection basins using the two FWD configurations. As described before, the curvature of the convex deflection basin between the loads provided a measurement of near surface layer stiffness. Therefore, configurations that provided a more distinct curvature in the deflection basins were considered superior in discrimining the effect of the stiffness.

Figures 8.1 to 8.3 illustrate comparisons of the dual-load-rigid-plate FWD deflection basin generated by the RIGID program for different load spacing and the effect of load spacing on surface curvature. Figures 8.1 and 8.2 indicate that each load spacing produced a apparent concave transverse deflection on the low asphalt concrete layer modulus. For a very thick and stiff pavement section (Figure 8.3), a slight advantage was observed when a spacing of 40-inch was used. Therefore, a wider load spacing (40 in.) can provide superior discrimination of the effect of the near surface layers, especially for very stiff and thick pavements.

However, the curvature of the deflection basin in the longitudinal direction is closely related to the stiffness of the lower pavement layer, such as subbase course. Figures 8.1 to 8.3 indicate that a spacing of 40-inch produce a flatter convex curvature in the longitudinal direction than a narrow spacing. This reduces the potential effect for lower pavement section moduli on the longitudinal deflection basin. In this case, the multi-layer moduli have a higher degree of interaction such that the subbase course modulus is barely

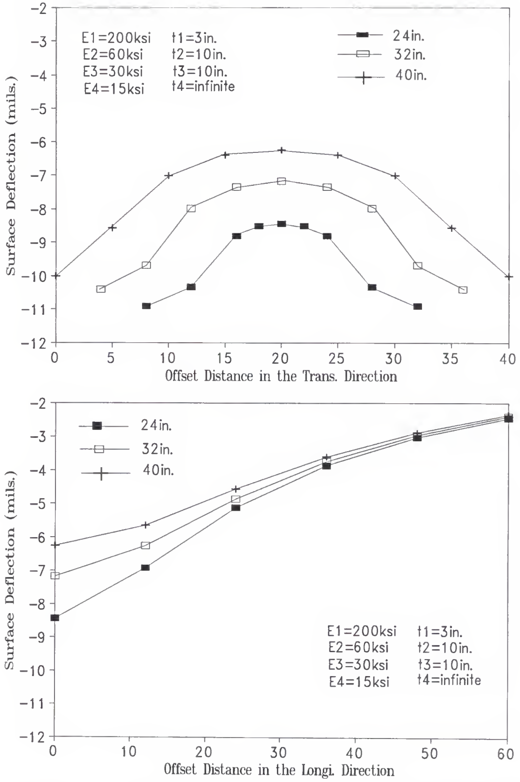


Figure 8.1 Comparison of Deflection Basins Induced By Different Load Spacing on a Thin Pavement Section

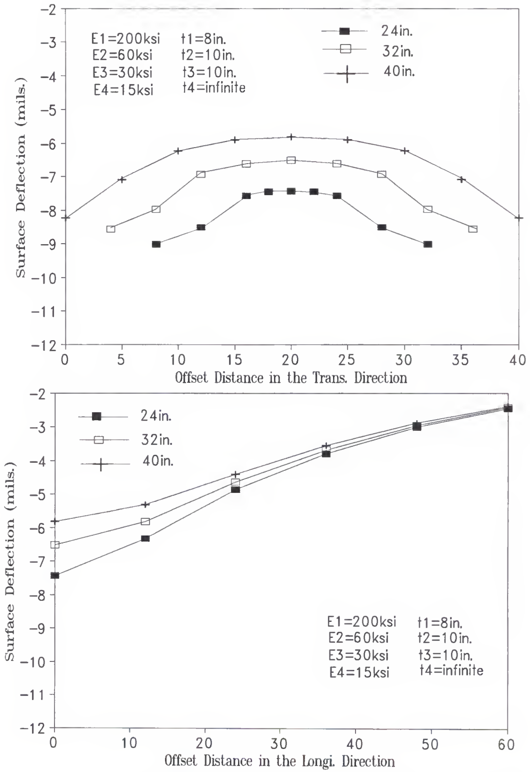


Figure 8.2 Comparison of Deflection Basins Induced By Different Load Spacing on a Thick Pavement Section

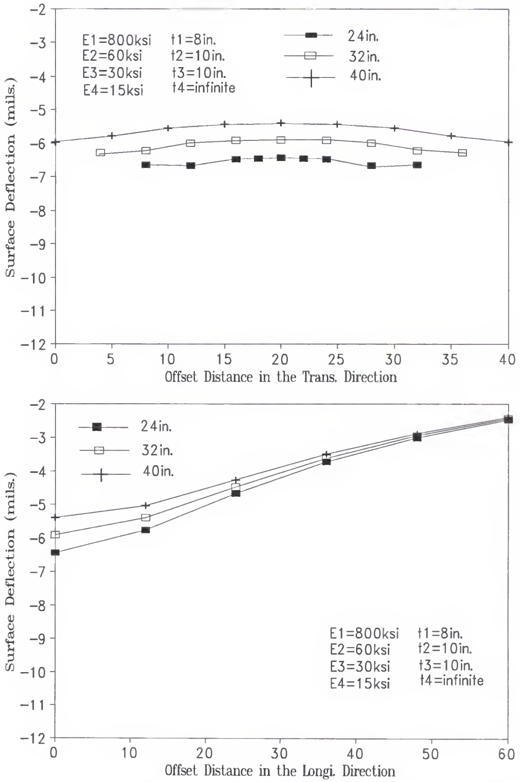


Figure 8.3 Comparison of Deflection Basins Induced By Different Load Spacing on a Stiff and Thick Pavement Section

noticeable. In conclusion, a dual-load-rigid-plate system with a wider load spacing can provide superior discrimination for evaluation of stiffness near the pavement surface. Also, it produces a strong interaction between multi-layer moduli and the surface deflections in the longitudinal direction, which decreases the ability for discrimination of the stiffness of the lower pavement section. Therefore, a dual-load-rigid-plate FWD system with the 24-inch load spacing provided superior discrimination for thin pavement sections or thicker pavement sections with lower layer moduli. The dual-load-rigid-plate FWD system with 40-inch load spacing was good for very thick and stiff pavement section. However, due to the wider load spacing used, the predictions of subbase course modulus (E_3) values using the dual-load-rigid-plate FWD with 40-inch load spacing is affected by other layer moduli, consequently, no direct prediction equation was developed.

8.3 The Adaptation of Standard FWD System

One of the major problems associated with the dual-rigid-load is that FWD equipment of this type does not exists at this time. Therefore, field tests can not be performed for a direct evaluation of the system. In accordance with the linear elastic theory, which implies that deformations (or deflections) are proportional to the loads applied to the medium or media, the standard single load FWD system can be used to simulate a dual-load system by superposition. This

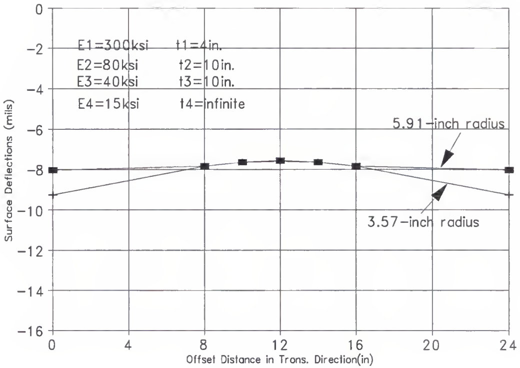
section will illustrate this adaptation of the FWD sensor configuration. In addition, a set of tentative layer-modulus prediction equations were developed for the 11.82 inch diameter FWD load plate to simulate this system.

8.3.1 Adaptation of the Standard FWD Configuration

In order to establish the sensor configuration of the adaptation of the standard FWD system, analyses were performed on two different load plate radii at two extreme pavement thicknesses and surface layer stiffness. Figures 8.4 and 8.5 show the deflection basins for two different load radii of 3.57 and 5.91 inches, respectively. As can be seen in both figures, the radius of the load plate has a very limited effect on the shape of the deflection basin except the deflections under the loads. This appeared to indicate that the sensor deflections relating to the specific layer-moduli in the two dual-load-rigid-plate system would be suitable for layer moduli predictions using the adaptation FWD system. Therefore, the sensor positions were selected from the analytical study on the basis of being related to the moduli of specific layers for the two dual-load-rigid-plate FWD systems (24 and 40 in. load-spacing). Figure 3.5 shows this adaptation of the standard FWD system.

8.3.2 Development of Layer Moduli Prediction Equations

The RIGID program was used to generate FWD deflection basins for the FWD adaptation configuration using different combinations of moduli and thicknesses covering the range



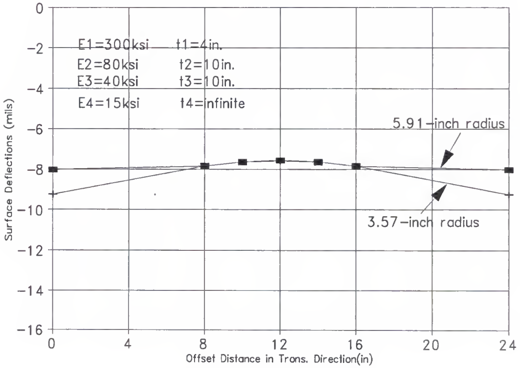
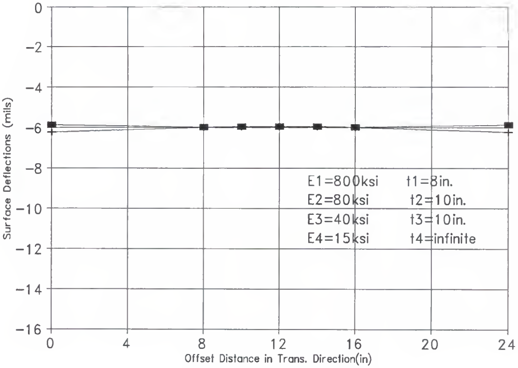


Figure 8.4 Comparison of Deflection Basins Induced by Different Load Radii on a 4-inch Pavement Section



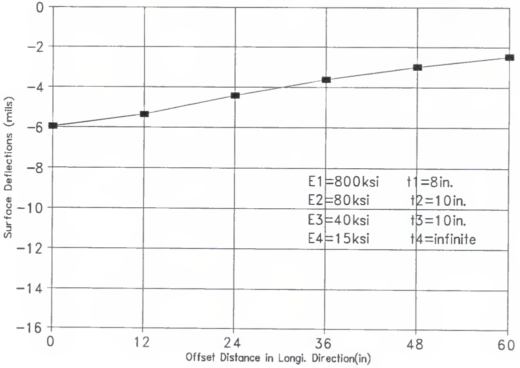


Figure 8.5 Comparison of Deflection Basins Induced By Different Load Radii on a 8-inch Pavement Section

listed in Table 5.3. Two sets of tentative prediction equations were developed based on previous study. One was used to simulate the dual-rigid-load FWD with 24-inch load spacing. The other one was for simulating the dual-load-rigid-plate FWD with 40-inch load spacing. The SAS program was used to perform all regression analyses. The application programming and program output are illustrated in Appendix C. The summary of results from regression analyses are described below:

A. Adaptation of Standard FWD System with 11.82 in. Diameter Plates at 24 in. Load Spacing (8,000 lbs total)

1. Asphalt concrete layer moduli E_1:

$$E_1 = 78.2254(t_1)^{0.5554}(D_{y/0} - D_{y/10})^{(-0.7966-19.1332/t1)} * (D_{y/0} - D_{x/8})^{17.4791/t1} \qquad \text{Eqn. 8.1}$$

$R^2 = 0.968$

2. Layer moduli E_{12} (composite of E1 and E2):

$$E_{12} = 1016.066(t_1)^{-0.5286}(t_2)^{-0.4085}[(D_{y/0} + D_{y/24})/2 - 0.985D_{x/8})^{-1.0202}$$

$R^2 = 0.955$ Eqn. 8.2

where $E_{12} = (E_1t_1 + E_2t_2)/(t_1 + t_2)$

3. Subgrade Moduli E_4:

$E_4 = 36.334\ (D_{x/60})^{-1.015}$ $R^2 = 0.996$ Eqn. 8.3

4. Subbase Moduli E_3:

$$E_3 = 105.81136(t_2)^{-1.0785}(D_{x/36} - D_{x/60})^{-6.02523 + 2.4888/(Dx/60)} * [(D_{y/0} + D_{x/12}) - 1.15D_{x/36}]^{2.1609 - 1.6202/(Dx/60) - 5.302/t2} * (D_{x/60})^{3.6706 + -0.0498t1 - 0.0686t2 - 3.0998/(Dx/60)} \qquad \text{Eqn. 8.4}$$

$R^2 = 0.904$

B. Adaptation of Standard FWD System with 11.82 in. Diameter Plates at 40 in. Load Spacing (8,000 lbs total)

1. Asphalt concrete layer moduli E_1:

$$E_1 = 281.2432(D_{y/0}-D_{y/10})^{(-0.8601-9.364/t1)}(D_{y/0} - D_{x/12})^{7.5662/t1}$$

$$R^2 = 0.953 \qquad \text{Eqn. 8.5}$$

2. Layer moduli E_{12} (composite of E1 and E2):

$$E_{12} = 4374.365(t_1)^{-0.8372}t_2^{-0.4666}(D_{y/0} - 1.03D_{y/10})^{-7.1512/t1}$$

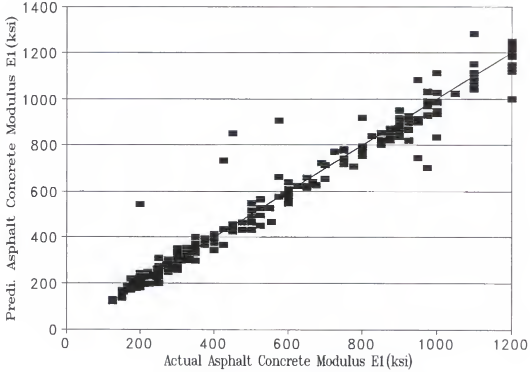
$$[(D_{y/0} + D_{y/40})/2 - D_{x/12}]^{-0.9282+6.4374/t1} \qquad \text{Eqn. 8.6}$$

$$R^2 = 0.967$$

where $E_{12} = (E_1t_1 + E_2t_2)/(t_1 + t_2)$

These results indicated that the regression models (formations) for the adaptation of the standard FWD system were similar to those previously developed for the FWD dual load systems. These equations can be evaluated by the field test, which can indirectly assess the relative accuracy of the FWD dual-load-rigid-plate system.

The residual analysis shown in Figures 8.6 to 8.11 do not reveal anything particularly troublesome, although some large residuals do stand out somewhat from the others. However, most prediction and actual values match fairly well. No severe problem was observed; therefore, these regression functions were considered acceptable.



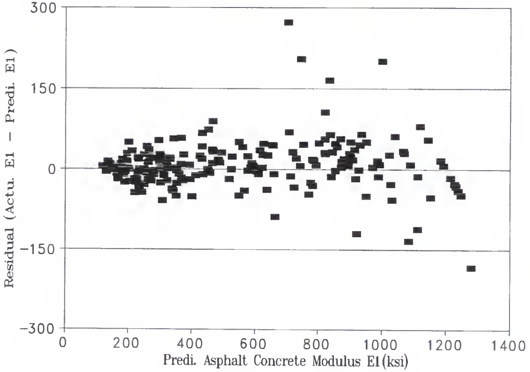
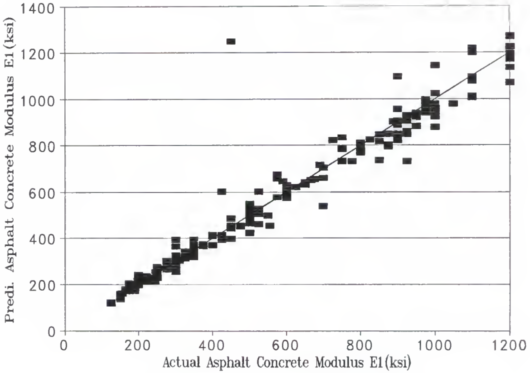


Figure 8.6 Residuals versus Predicted E_1 Values (Simulating 24-inch Load Spacing)



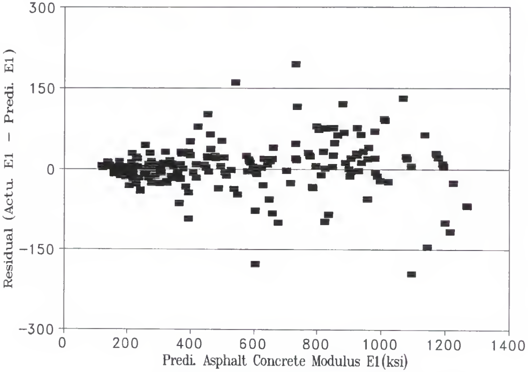
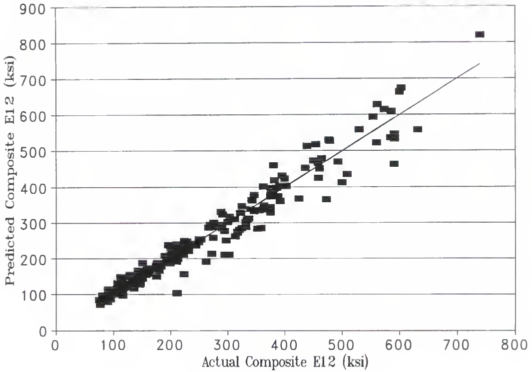


Figure 8.7 Residuals versus Predicted E_1 Values (Simulating 40-inch load Spacing)



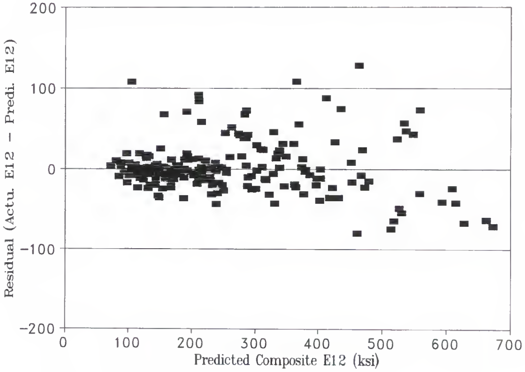
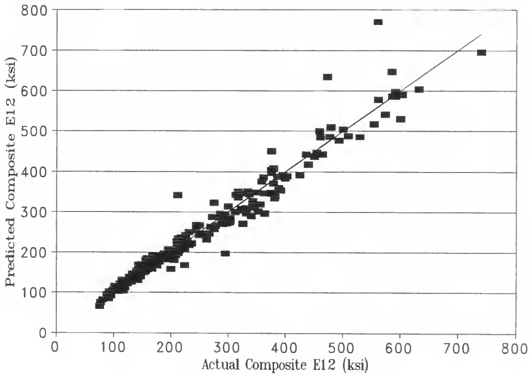


Figure 8.8 Residuals versus Predicted E_{12} Values (Simulating 24-inch Load Spacing)



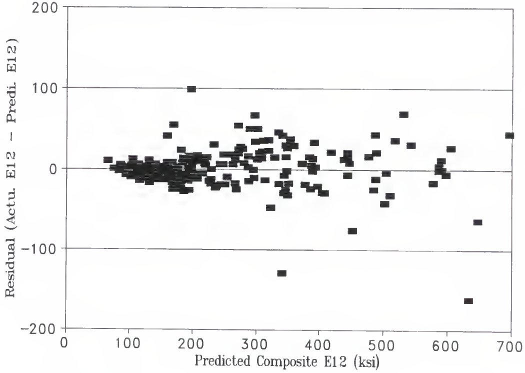


Figure 8.9 Residuals versus Predicted E_{12} Values (Simulating 40-inch Load Spacing)

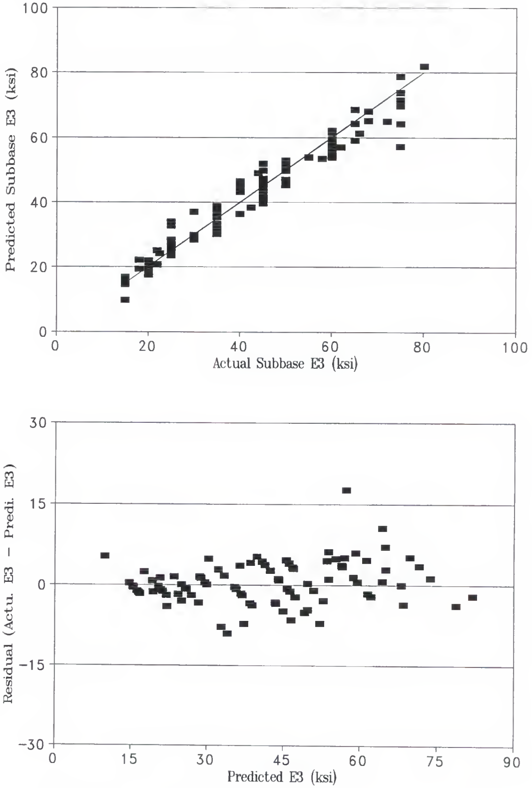
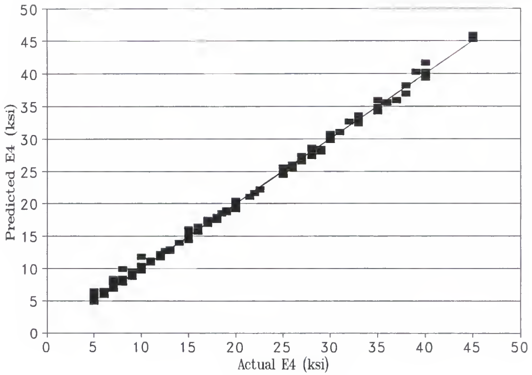


Figure 8.10 Residuals versus Predicted E_3 Values (Simulating 24-inch Load Spacing)



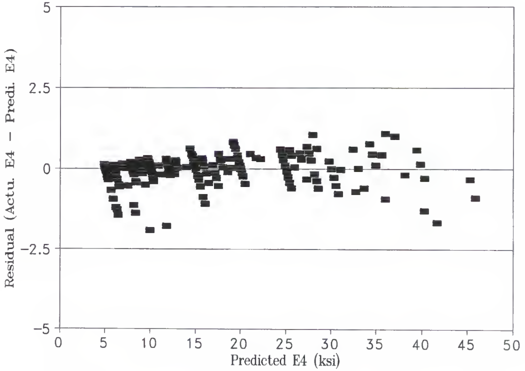


Figure 8.11 Residuals versus Predicted E_4 Values (Simulating 24-inch Load Spacing)

CHAPTER 9
CONCLUSIONS AND RECOMMENDATIONS

9.1 Conclusions

The primary objective of this study was to develop layer moduli prediction procedures using a dual-load-rigid-plate Falling Weight Deflectometer (FWD). A set of moduli prediction equations was developed for three different FWD configurations covering specific ranges in layer thickness and layer modulus. These equations provided the basis for prediction of layer moduli using dual-load-rigid-plate FWD systems. It should be pointed out that the prediction equations obtained in this study are only applicable to the range of parameters listed in Tables 5.1 through 5.3.

Some researchers suggested that the dual-load system with wide load spacing would result in superior discrimination for evaluation of stiffness of the pavement. The findings from this study disagree with this suggestion in the case of the subbase layer moduli E_3.

It should be emphasized that this investigation was based only upon the results of analytical studies. No field tests were performed for the direct evaluation of the developed layer moduli prediction equations. The following conclusions can be drawn from the results obtained in this investigation:

1. Double bending FWD systems provide more information to discriminate the individual effects of all pavement layer moduli than the existing single-load FWD.

2. The curvature of the convex deflection basin between the loads was strongly and almost uniquely related to the asphalt concrete layer modulus. The base course, the subbase course and subgrade moduli affected only the magnitude of the deflections in the transverse direction, but not the shape of the deflection basin.

3. The curvature of the deflection basin in the longitudinal direction is closely related to the stiffness of the lower pavement layers, such as subbase course and subgrade. In addition, the changes in E_1, E_2, and E_3 did not have a significant effect on the deflections over 60 inches in the longitudinal direction. Therefore, this sensor deflection ($D_{x/60}$) was uniquely related to the subgrade modulus E_4.

4. Regression analysis results of the relationship between pavement moduli and deflections and/or deflection differences provided a reasonable means of estimating pavement moduli from deflections produced by dual-load-rigid-plate FWD systems. It was observed that the developed predictive equations were fairly accurate in providing an estimate of pavement layer moduli.

5. Analytical evaluation of the dual-load-rigid-plate FWD system with a 24-inch load spacing indicated that the asphalt concrete modulus E_1, combined modulus E_{12} for asphalt concrete

E_1 and base course E_2 , subbase course modulus E_3 and subgrade modulus E_4 could be predicted based on pavement thicknesses and deflection basin characteristics measured by using the unique load sensor configuration.

6. The base course moduli (E_2) can be obtained from the weighting equation if the composite modulus (E_{12}) is calculated upon Barro's approximation formula ($E_{12}^{1/3} = [E_1^{1/3}t_1 + E_2^{1/3}t_2]/[t_1 + t_2]$). However, this indirect method may lead to substantial prediction errors. Therefore, the reliability of E_2 predictions for the 24-inch load spacing should be checked using iteration or available back calculation techniques.

7. The dual-load-rigid-plate FWD system with 40-inch load spacing can be used to determine (or predict) the asphalt concrete modulus E_1, composite modulus E_{12}, and subgrade modulus E_4. Base course modulus E_2 can be predicted through the weighting equation. However, no relationships were identified that could be used for subbase moduli predictions.

8. In general, a dual-load-rigid-plate FWD system with the 24-inch load spacing provided superior discrimination for thin pavements. The dual-rigid-load FWD system with 40-inch load spacing was good for very thick and stiff pavements. However, it produced a strong interaction between other layer moduli (E_2, E_3 and E_4) which affected the surface deflections in the longitudinal direction.

9. The radius of the load plate does not have an apparent effect on the curvature of the deflection basins.

10. The adaptation of the standard FWD system provides a very useful method for direct evaluation of the dual-load system.

9.2 Recommendations

The techniques developed in this investigation eliminate many of the problems associated with current methods in the determination of realistic pavement layer moduli. Therefore, the most important recommendation is to actually construct a dual-load Falling Weight Deflectometer with adjustable load spacing for field testing of pavement systems using the developed equations to predict layer moduli and assess structural adequacy. In addition, the following recommendations for further study are suggested:

1. Field tests should be carried out for the direct evaluation of the dual load FWD systems and verification of the accuracy of the developed prediction equations.
2. The algorithms obtained in this study should be incorporated into a computer program such as REDAPS for mechanistic evaluation and rehabilitation design of flexible pavements.
3. Additional studies are required to develop the direct prediction equation of base course moduli (E_2) from the surface deflections.
4. The comparative analysis of 24 and 40 inch dual-load spacings indicated that these spacings have different capabilities for discrimination of layer moduli. Therefore,

it is recommended that additional work be conducted to develop optimal load spacings which will provide good layer discrimination for various pavement layer thicknesses and stiffnesses.

APPENDIX A
SAS PORGRAMING FOR DUAL-LOAD-RIGID-PLATE FWD WITH 24-INCH LOAD SPACING

SAS programing input for E_1 prediction

```
options ps=60;
data shen;
infile 'c:\fwd-24\e1-24.dat';
input E1 E2 E3 E4 t1 t2 t3 D0 D08 D00 D8 D12 D20 D24 D36 D48
D60 x5;
y=log10(E1);
x1=D60;
x2=D0-0.975*D08;
x3=D0-D8;
x4=D48-D60;
proc sysnlin method=marquardt;
endo y;
exo t1 t2 x1 x2 x3;
y=(A0+A1*log10(t1))
+(B0+B1/t1)*log10(x2);
fit y start=(A0 -20 A1 2 A2 0 A3 0.0 A4 0.1
B0 1.1 B1 0 B2 -0.02 B3 0 B4 0.3
C0 -6.0 C1 -0.1 C2 0.0 C3 -1.2 C4 0
D01 5 D1 -7 D2 -3.0 D3 -1.0 D4 0 E0 -0.5 E1 0 E2 1);
Parms A0 A1 A2 A3 B0 B1 B2 B3 C0 C1 C2 C3 C4 D0 D1 D2;
run;
```

SAS programing output for E_1 prediction

```
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                SYSNLIN Procedure

                 Model Summary

          Model Variables          6
          Endogenous               1
          Exogenous                5
          Parameters              24
          Equations                1

                          Number of Statements    1

          Model Variables: Y T1 T2 X1 X2 X3

Parameters: A0: -20  A1: 2  B0: 1.1  B1: 0  A2: 0  A3: 0
            B2: -0.02  B3: 0

          Equations: Y

 SAS    3:23 Saturday, August 22, 1992  34

             SYSNLIN Procedure

      The Equation to Estimate is:

      Y = F( A0(1), A1, B0, B1 )

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```

SYSNLIN Procedure
OLS Estimation

OLS Estimation Summary

Dataset Option	Dataset
DATA=	SHEN

Parameters Estimated	4

Minimization Summary

Method	MARQUARDT
Iterations	1

Final Convergence	Criteria
R	0.00023418
PPC(B1)	0.000208
RPC(B1)	1338073
Object	0.9999889
Trace(S)	0.00512011
Objective Value	0.00503979
Lambda	1E-7

Observations Processed	
Read	255
Solved	255

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SYSNLIN Procedure
OLS Estimation

Nonlinear OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
Y	4	251	1.28515	0.0051201	0.07155	0.9342	0.9334

Nonlinear OLS Parameter Estimates

Parameter	Estimate	Approx. Std Err	'T' Ratio	Approx. Prob>|T|
A0	3.351546	0.02919	114.80	0.0001
A1	-0.991072	0.03957	-25.05	0.0001
B0	-0.778824	0.05070	-15.36	0.0001
B1	-1.338073	0.26464	-5.06	0.0001

Number of Observations		Statistics for System	
Used	255	Objective	0.005040
Missing	0	Objective*N	1.2851

SAS programing input for E_{12} prediction

```
options ps=60;
data shen;
infile 'c:\fwd-24\fwd24.dat';
input E1 E2 E3 E4 t1 t2 t3 D0 D8 D12 D20 D24 D36 D48 D60;
E12=(E1*t1+E2*t2)/(t1+t2);
y=log10(E12);
proc sysnlin method=marquardt;
endo y;
exo t1 t2 D0 D8;
y=B0+B1*log10(t1)+B2*log10(t2)+(B3+B4*t1)*log10(D0-B5*D8);
fit y start=(B0 3 B1 -1.5 B2 0.0 B3 -2.0 B4 0 B5 -2.0);
Parms B0 B1 B2 B3 B4 B5;
run;
```

SAS programing output for E_{12} prediction

```
  SAS   17:29 Thursday, August 13, 1992   7

            SYSNLIN Procedure

               Model Summary

         Model Variables          5
         Endogenous               1
         Exogenous                4
         Parameters               6
         Equations                1

         Number of Statements     1

     Model Variables: Y T1 T2 D0 D8

Parameters: B0: 3  B1: -1.5  B2: 0  B3: -2  B4: 0  B5: -2

          Equations: Y

   SAS   17:29 Thursday, August 13, 1992   8

            SYSNLIN Procedure

        The Equation to Estimate is:

       Y = F( B0(1), B1, B2, B3, B4, B5 )

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            SYSNLIN Procedure
               OLS Estimation

          OLS Estimation Summary

          Dataset Option       Dataset
          DATA=                   SHEN
```

Parameters Estimated 6

Minimization Summary	
Method	MARQUARDT
Iterations	8
Subiterations	9
Average Subiterations	1.12

Final Convergence Criteria	
R	9.89002E-6
PPC(B2)	7.956E-6
RPC(B2)	0.00241
Object	2.52535E-6
Trace(S)	0.0026841
Objective Value	0.00265913
Lambda	1E-5

Observations Processed	
Read	645
Solved	645

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SYSNLIN Procedure
OLS Estimation

Nonlinear OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
Y	6	639	1.71514	0.0026841	0.05181	0.9612	0.9609

Nonlinear OLS Parameter Estimates

Parameter	Estimate	Approx. Std Err	'T' Ratio	Approx. Prob>|T|
B0	3.358291	0.02945	114.04	0.0
B1	-0.643140	0.02060	-31.23	0.0001
B2	-0.369645	0.02324	-15.90	0.0001
B3	-1.292827	0.02966	-43.59	0.0001
B4	0.053280	0.0043716	12.19	0.0001
B5	0.989517	0.0012420	796.74	0.0

Number of Observations	
Used	645
Missing	0

Statistics for System	
Objective	0.002659
Objective*N	1.7151

SAS programing input for E_3 prediction

```
options ps=60;
data shen;
infile 'c:\fwd-24\e3-24.dat';
input E1 E2 E3 E4 t1 t2 t3 D0 D08 D00 D8 D12 D20 D24 D36 D48
D60;
y=log10(E3);
x1=D60;
x2=D0-1.1*D24;
x3=D36-D60;
proc sysnlin method=marquardt;
endo y;
exo t1 t2 x1 x2 x3;
y=(A0+A2*log10(t2))
+(B0+B1/x1+B3/t2)*log10(x2)
+(C0)*log10(x3)
+(D01+D1/t1+D2/t2)*log10(x1);
fit y start=(A0 -20 A1 2 A2 0 A3 0.0 A4 0.1 B0 1.1 B1 0 B2
-0.02 B3 0 B4 0.3 C0 -6.0 C1 -0.1 C2 0.0 C3 -1.2 C4 0
D01 5 D1 -7 D2 -3.0 D3 -1.0);
Parms A0 A1 A2 A3 B0 B1 B2 B3 C0 C1 C2 C3 C4 D0 D1 D2;
run;
```

SAS programing output for E_3 prediction

```
             SYSNLIN Procedure

               Model Summary

          Model Variables         6
          Endogenous              1
          Exogenous               5
          Parameters             24
          Equations               1

          Number of Statements    1

     Model Variables: Y T1 T2 X1 X2 X3

Parameters: A0: -20  A2: 0 B0: 1.1 B1: 0  B3: 0 C0: -6  D01:5
           D1: -7  D2: -3  A1: 2  A3: 0  A4: 0.1  B2: -0.02
           B4: 0.3  C1: -0.1  C2: 0  C3: -1.2  C4: 0  D3: -1

          Equations: Y

               SYSNLIN Procedure

        The Equation to Estimate is:

    Y = F( A0(1), A2, B0, B1, B3, C0, D01, D1, D2 )

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                   SYSNLIN Procedure
                   OLS Estimation
```

OLS Estimation Summary

Dataset Option	Dataset
DATA=	SHEN

Parameters Estimated	9

Minimization Summary

Method	MARQUARDT
Iterations	2

Final Convergence	Criteria
R	4.02864E-8
PPC(B3)	9.613E-8
RPC(B3)	0.003315
Object	2.11452E-6
Trace(S)	0.00333682
Objective Value	0.00321905
Lambda	1E-8

Observations Processed

Read	255
Solved	255

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SYSNLIN Procedure
OLS Estimation

Nonlinear OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
Y	9	246	0.82086	0.0033368	0.05777	0.9413	0.9394

Nonlinear OLS Parameter Estimates

Parameter	Estimate	Approx. Std Err	'T' Ratio	Approx. Prob>|T|
A0	1.930264	0.13079	14.76	0.0001
A2	-1.088873	0.12898	-8.44	0.0001
B0	1.215081	0.11429	10.63	0.0001
B1	-0.724488	0.05439	-13.32	0.0001
B3	-4.918579	1.07867	-4.56	0.0001
C0	-4.058075	0.07298	-55.60	0.0001
D01	1.763999	0.09850	17.91	0.0001
D1	0.807712	0.13435	6.01	0.0001
D2	3.968265	0.93326	4.25	0.0001

Number of Observations	
Used	255
Missing	0

Statistics for System	
Objective	0.003219
Objective*N	0.8209

SAS programing input for E_4 prediction

```
options ps=60;
data shen;
infile 'c:\fwd-24\fwd24.dat';
input E1 E2 E3 E4 t1 t2 t3 D0 D8 D12 D20 D24 D36 D48 D60;
y=log10(E4);
proc sysnlin method=marquardt;
endo y;
exo t1 t2 D48;
y=B0+B1*log10(D48);
fit y start=(B0 1 B1 -1.5);
Parms B0 B1;
run;
```

SAS programing output for E_4 prediction

```
   SAS   18:46 Thursday, August 13, 1992   5

                 SYSNLIN Procedure

                 Model Summary

                 Model Variables         5
                 Endogenous              1
                 Exogenous               4
                 Parameters              2
                 Equations               1

                 Number of Statements    1

         Model Variables: Y T1 T2 D48 D60

         Parameters: B0: 3  B1: -1.5

                  Equations: Y

         SAS   18:46 Thursday, August 13, 1992   6

                  SYSNLIN Procedure

              The Equation to Estimate is:

                  Y = F( B0(1), B1 )

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                    SYSNLIN Procedure
                    OLS Estimation
```

OLS Estimation Summary

Dataset Option	Dataset
DATA=	SHEN

Parameters Estimated	2

Minimization Summary

Method	MARQUARDT
Iterations	1

Final Convergence	Criteria
R	9.36149E-6
PPC(B1)	5.488E-7
RPC(B0)	0.446136
Object	0.99948289
Trace(S)	0.00068785
Objective Value	0.00068572
Lambda	1E-7

Observations Processed

Read	645
Solved	645

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SYSNLIN Procedure
OLS Estimation

Nonlinear OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
Y	2	643	0.44229	0.0006879	0.02623	0.9951	0.9951

Nonlinear OLS Parameter Estimates

Parameter	Approx. Estimate	Std Err	'T' Approx. Ratio	Prob>|T|
B0	1.661592	0.0016393	1013.58	0.0
B1	-1.050344	0.0028985	-362.38	0.0

Number of Observations	
Used	645
Missing	0

Statistics for System	
Objective	0.000686
Objective*N	0.4423

SAS programing input for E_4 prediction

```
options ps=60;
data shen;
infile 'c:\fwd-24\fwd24.dat';
input E1 E2 E3 E4 t1 t2 t3 D0 D8 D12 D20 D24 D36 D48 D60;
y=log10(E4);
proc sysnlin method=marquardt;
endo y;
exo t1 t2 D60;
y=B0+B1*log10(D60);
fit y start=(B0 1 B1 -1.5);
Parms B0 B1;
run;
```

SAS programing output for E_4 prediction

```
       SAS   19:04 Thursday, August 13, 1992   1

                 SYSNLIN Procedure

                  Model Summary

              Model Variables         4
              Endogenous              1
              Exogenous               3
              Parameters              2
              Equations               1

              Number of Statements    1

              Model Variables: Y T1 T2 D60

              Parameters: B0: 1  B1: -1.5

                 Equations: Y

      SAS   19:04 Thursday, August 13, 1992   2

                  SYSNLIN Procedure

            The Equation to Estimate is:

                  Y = F( B0(1), B1 )

      SAS   19:04 Thursday, August 13, 1992   3

                   SYSNLIN Procedure
                   OLS Estimation
```

OLS Estimation Summary

Dataset Option	Dataset
DATA=	SHEN

Parameters Estimated 2

Minimization Summary

Method	MARQUARDT
Iterations	1

Final Convergence	Criteria
R	3.32457E-6
PPC(B1)	4.749E-8
RPC(B0)	0.556892
Object	0.99945649
Trace(S)	0.00031017
Objective Value	0.00030921
Lambda	1E-7

Observations Processed

Read	644
Solved	644

SAS 19:04 Thursday, August 13, 1992 4

SYSNLIN Procedure
OLS Estimation

Nonlinear OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
Y	2	642	0.19913	0.0003102	0.01761	0.9978	0.9978

Nonlinear OLS Parameter Estimates

Parameter	Estimate	Approx. Std Err	'T' Ratio	Approx. Prob>|T|
B0	1.556892	0.0009574	1626.18	0.0
B1	-1.004675	0.0018618	-539.62	0.0

Number of Observations	
Used	644
Missing	0

Statistics for System	
Objective	0.000309
Objective*N	0.1991

SAS programing input for E_1 prediction (FINAL)

```
options ps=60;
data shen;
infile 'c:\fwd-24\e1-24.dat';
input E1 E2 E3 E4 t1 t2 t3 D0 D08 D00 D8 D12 D20 D24 D36 D48
D60 x5;
y=log10(E1);
x2=D0-0.97*D08;
x3=D0-D8;
proc sysnlin method=marquardt;
endo y;
exo t1 t2 x2 x3;
y=(A0)
+(B0+B1/t1)*log10(x2)
+(C0+C1/t1)*log10(x3);
fit y start=(A0 -20 A1 2 A2 0 A3 0.0 A4 0.1
B0 1.1 B1 0 B2 -0.02 B3 0 B4 0.3
C0 -6.0 C1 -0.1 C2 0.0 C3 -1.2 C4 0);
Parms A0 A1 A2 A3 B0 B1 B2 B3 C0 C1 C2 C3 C4;
run;
```

SAS programing output for E_1 prediction (FINAL)

```
       SAS   8:19 Wednesday, August 12, 1992 106

                  SYSNLIN Procedure

                    Model Summary

                Model Variables        6
                Endogenous             1
                Exogenous              5
                Parameters            24
                Equations              1

                Number of Statements   1

          Model Variables: Y T1 T2 X2 X3

Parameters: A0: -20  B0: 1.1  B1: 0  C0: -6  C1: -0.1  A1: 2
            A2: 0  A3: 0  A4: 0.1  B2: -0.02  B3: 0  B4: 0.3
            C2: 0  C3: -1.2  C4: 0

          Equations: Y

                  SYSNLIN Procedure

             The Equation to Estimate is:

             Y = F( A0(1), B0, B1, C0, C1 )

         SAS   8:19 Wednesday, August 12, 1992 108
```

SYSNLIN Procedure
OLS Estimation

OLS Estimation Summary

Dataset Option	Dataset
DATA=	SHEN

Parameters Estimated	5

Minimization Summary

Method	MARQUARDT
Iterations	1

Final Convergence	Criteria
R	0.00016038
PPC(B0)	0.000139
RPC(B1)	13282572
Object	0.99999612
Trace(S)	0.002186
Objective Value	0.00214314
Lambda	1E-7

Observations Processed

Read	255
Solved	255

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SYSNLIN Procedure
OLS Estimation

Nonlinear OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
Y	5	250	0.54650	0.0021860	0.04675	0.9720	0.9716

Nonlinear OLS Parameter Estimates

Parameter	Estimate	Approx. Std Err	'T' Ratio	Approx. Prob>|T|
A0	2.646018	0.00977	270.95	0.0
B0	1.589290	0.10580	15.02	0.0001
B1	-13.282572	0.40400	-32.88	0.0001
C0	-2.453435	0.09773	-25.10	0.0001
C1	12.233590	0.31299	39.09	0.0001

Number of Observations		Statistics for System	
Used	255	Objective	0.002143
Missing	0	Objective*N	0.5465

SAS programing input for E_{12} prediction with Barros' weighting equation

```
options ps=60;
data shen;
infile 'c:\fwd-24\fwd24.dat';
input E1 E2 E3 E4 t1 t2 t3 D0 D8 D12 D20 D24 D36 D48 D60;
E12=((E1**(1/3)*t1+E2**(1/3)*t2)/(t1+t2))**3;
y=log10(E12);
x1=D0-D12;
x2=D0-1.03*D20;
proc sysnlin method=marquardt;
endo y;
exo t1 t2 D0 D12;
y=B0+(B3+B4/t1+B5/t2)*log10(x1)
+(B6+B7/t1+B8/t2)*log10(x2);
fit y start=(B0 3 B1 -1.5 B2 0.0 B3 -2.0 B4 0 B5 -2.0 B6 -1
B7 0.5 B8 0.5);
Parms B0 B1 B2 B3 B4 B5 B6 B7 B8;
run;
```

SAS programing output for E_{12} prediction with Barros' weighting equation

```
            SAS       1:34 Thursday, May 10, 1993  65

                    SYSNLIN Procedure

                      Model Summary

                 Model Variables        5
                 Endogenous             1
                 Exogenous              4
                 Parameters             9
                 Equations              1

                 Number of Statements   1

         Model Variables: Y T1 T2 D0 D12

Parameters: B0: 3  B3: -2  B4: 0  B5: -2  B6: -1  B7: 0.5
            B8: 0.5  B1: -1.5  B2: 0

         Equations: Y

        SAS       1:34 Thursday, July 2, 1992  66

                    SYSNLIN Procedure

              The Equation to Estimate is:

         Y = F( B0(1), B3, B4, B5, B6, B7, B8 )

        SAS       1:34 Thursday, July 2, 1992  67
```

SYSNLIN Procedure
OLS Estimation

OLS Estimation Summary

Dataset Option	Dataset
DATA=	SHEN
Parameters Estimated	7

Minimization Summary

Method	MARQUARDT
Iterations	1

Final Convergence	Criteria
R	0.00031009
PPC(B3)	0.000293
RPC(B4)	7017501
Object	0.9964609
Trace(S)	0.00135158
Objective Value	0.00133689
Lambda	1E-7

Observations Processed

Read	644
Solved	644

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SYSNLIN Procedure
OLS Estimation

Nonlinear OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
Y	7	637	0.86096	0.0013516	0.03676	0.9697	0.9695

Nonlinear OLS Parameter Estimates

Parameter	Estimate	Approx. Std Err	'T' Ratio	Approx. Prob>|T|
B0	2.562676	0.0084167	304.48	0.0
B3	2.505825	0.09508	26.36	0.0001
B4	-7.017501	0.25437	-27.59	0.0001
B5	-15.359230	0.65505	-23.45	0.0001
B6	-3.377776	0.08856	-38.14	0.0001
B7	6.514876	0.21186	30.75	0.0001
B8	15.433640	0.53489	28.85	0.0001

Number of Observations		Statistics for System	
Used	644	Objective	0.001337
Missing	0	Objective*N	0.8610

APPENDIX B
SAS PORGRAMING FOR DUAL-LOAD-RIGID-PLATE FWD WITH 40-INCH LOAD SPACING

SAS programing input for E_1 prediction
$t_1 + 0.2t_2 \leq 7$in.

```
options ps=60;
data shen;
infile 'c:\fwd40\fwd40-1.dat';
input E1 E2 E3 E4 t1 t2 t3 D0 D10 D15 D00 D8 D12 D20 D24 D36
D60;
y=log10(E1);
x1=D0-0.99*D00;
x2=D0-0.96*D10;
x3=D10-0.96*D00;
proc sysnlin method=marquardt;
endo y;
exo t1 t2 x1 x2 x3;
y=(A0+A1*log10(t1)+A2*log10(t2))
+(B0+B1/t1+B2/t2)*log10(x2)
+(C1/t1)*log10(x3)
+(D01+D1/t1+D2/t2)*log10(x1);
fit y start=(A0 -20 A1 2 A2 0 A3 0.0 A4 0.1
B0 1.1 B1 0 B2 -0.02 B3 0 B4 0.3
C0 -6.0 C1 -0.1 C2 0.0 C3 -1.2 C4 0
D01 5 D1 -7 D2 -3.0);
Parms A0 A1 A2 A3 B0 B1 B2 B3 C0 C1 C2 C3 C4 D0 D1 D2;
run;
```

SAS programing output for E_1 prediction
$t_1 + 0.2t_2 \leq 7$in.

```
                SAS   18:20 Saturday, August 15, 1992  62

                          SYSNLIN Procedure

                            Model Summary

                      Model Variables          6
                        Endogenous             1
                        Exogenous              5
                      Parameters              24
                      Equations                1

                      Number of Statements     1

          Model Variables: Y T1 T2 X1 X2 X3

Parameters: A0: -20  A1: 2  A2: 0  B0: 1.1  B1: 0  B2: -0.02
           C1: -0.1  D01: 5  D1: -7  D2: -3  A3: 0  A4: 0.1
           B3: 0  B4: 0.3  C0: -6  C2: 0  C3: -1.2  D3: -1
           D4: 0  E0: -0.5  E1: 0  E2: 1  D0

          Equations: Y

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                      SYSNLIN Procedure

                 The Equation to Estimate is:

       Y = F( A0(1), A1, A2, B0, B1, B2, C1, D01, D1, D2 )

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                     SYSNLIN Procedure
                       OLS Estimation

                   OLS Estimation Summary

                 Dataset Option        Dataset
                 DATA=                    SHEN

                 Parameters Estimated       10

                     Minimization Summary
                 Method              MARQUARDT
                 Iterations                  1

                 Final  Convergence  Criteria
                 R                  0.00075753
                 PPC(B2)              0.003709
                 RPC(B1)              16082507
                 Object             0.99999077
                 Trace(S)           0.00396349
                 Objective Value    0.00395464
```

Lambda	1E-7

Observations Processed

Read	4480
Solved	4480

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SYSNLIN Procedure
OLS Estimation

Nonlinear OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
Y	10	4470	17.71678	0.0039635	0.06296	0.9529	0.9528

Nonlinear OLS Parameter Estimates

Parameter	Estimate	Approx. Std Err	'T' Ratio	Approx. Prob>|T|
A0	3.576191	0.01694	211.12	0.0
A1	-1.110793	0.01570	-70.75	0.0
A2	0.313019	0.00971	32.24	0.0001
B0	1.287022	0.14284	9.01	0.0001
B1	-16.082507	0.33022	-48.70	0.0
B2	-2.980341	0.45050	-6.62	0.0001
C1	3.287027	0.02204	149.17	0.0
D01	-2.283218	0.12453	-18.33	0.0001
D1	12.649016	0.28554	44.30	0.0
D2	3.066172	0.39953	7.67	0.0001

Number of Observations

Used	4480
Missing	0

Statistics for System

Objective	0.003955
Objective*N	17.7168

SAS programing input for E_1 prediction
$t_1 + 0.2t_2 > 7$in.

```
options ps=60;
data shen;
infile 'c:\fwd40\fwd40-2.dat';
input E1 E2 E3 E4 t1 t2 t3 D0 D10 D15 D00 D8 D12 D20 D24 D36
D60;
y=log10(E1);
x1=D0-0.97*D10;
x2=D0-D12;
proc sysnlin method=marquardt;
endo y;
exo t1 t2 x1 x2;
y=(A0)
+(B0+B1/t1+B2/t2)*log10(x1)
+(C0+C1/t1+C2/t2)*log10(x2);
fit y start=(A0 -20 A1 2 A2 0 A3 0.0 A4 0.1
B0 1.1 B1 0 B2 -0.02 B3 0 B4 0.3
C0 -6.0 C1 -0.1 C2 0.0 C3 -1.2 C4 0
D01 5 D1 -7 D2 -3.0 D3 -1.0 D4 0);
Parms A0 A1 A2 A3 B0 B1 B2 B3 C0 C1 C2 C3 C4 D0 D1 D2;
run;
```

SAS programing output for E_1 prediction
$t_1 + 0.2t_2 > 7$in.

```
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                              SYSNLIN Procedure

                                Model Summary

                          Model Variables         5
                            Endogenous            1
                            Exogenous             4
                          Parameters             24
                          Equations               1

                          Number of Statements    1

          Model Variables: Y T1 T2 X1 X2

Parameters: A0: -20  B0: 1.1  B1: 0  B2: -0.02  C0: -6
            C1: -0.1 C2: 0  A1: 2  A2: 0  A3: 0  A4: 0.1
            B3: 0  B4: 0.3  C3: -1.2  C4: 0  D01: 5
            D1: -7  D2: -3  D3: -1  D4: 0

             Equations: Y

           SAS    2:14 Saturday, August 15, 1992  94

                     SYSNLIN Procedure

                 The Equation to Estimate is:

              Y = F( A0(1), B0, B1, B2, C0, C1, C2 )

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```

SYSNLIN Procedure
OLS Estimation

OLS Estimation Summary

Dataset Option	Dataset
DATA=	SHEN

Parameters Estimated	7

Minimization Summary

Method	MARQUARDT
Iterations	1

Final Convergence Criteria

R	0.00009973
PPC(B0)	0.000184
RPC(B1)	15575986
Object	0.99999682
Trace(S)	0.00169073
Objective Value	0.0016896
Lambda	1E-7

Observations Processed

Read	10528
Solved	10528

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SYSNLIN Procedure
OLS Estimation

Nonlinear OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
Y	7	10521	17.78812	0.0016907	0.04112	0.9799	0.9799

Nonlinear OLS Parameter Estimates

Parameter	Estimate	Approx. Std Err	'T' Ratio	Approx. Prob>|T|
A0	2.627628	0.0008378	3136.19	0.0
B0	0.495156	0.01843	26.87	0.0001
B1	-15.575986	0.10542	-147.76	0.0
B2	2.372871	0.04762	49.83	0.0
C0	-1.472057	0.01741	-84.55	0.0
C1	14.690078	0.09662	152.04	0.0
C2	-1.973322	0.04762	-41.44	0.0

Number of Observations	
Used	10528
Missing	0

Statistics for System	
Objective	0.001690
Objective*N	17.7881

SAS programing input for E_{12} prediction
$t_1 + 0.2t_2 \leq 7$in.

```
options ps=60;
data shen;
infile 'c:\fwd40\fwd40-1.dat';
input E1 E2 E3 E4 t1 t2 t3 D0 D10 D15 D00 D8 D12 D20 D24 D36
D60;
E12=(E1*t1+E2*t2)/(t1+t2);
y=log10(E12);
x1=D0-0.97*D00;
x2=D0-0.97*D10;
proc sysnlin method=marquardt;
endo y;
exo t1 t2 x1 x2 x3;
y=(A0+A1*log10(t1))
+(B0+B1/t1+B2/t2)*log10(x1)
+(C0+C1/t1+C2/t2)*log10(x2);
fit y start=(A0 -20 A1 2 A2 0 A3 0.0 A4 0.1
B0 1.1 B1 0 B2 -0.02 B3 0 B4 0.3
C0 -6.0 C1 -0.1 C2 0.0 C3 -1.2 C4 0
D01 5 D1 -7 D2 -3.0 D3 -1.0 D4 0);
Parms A0 A1 A2 A3 B0 B1 B2 B3 C0 C1 C2 C3 C4 D0 D1 D2;
run;
```

SAS programing output for E_{12} prediction
$t_1 + 0.2t_2 \leq 7$in.

```
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                            SYSNLIN Procedure

                              Model Summary

                        Model Variables         6
                          Endogenous            1
                          Exogenous             5
                        Parameters             24
                        Equations               1

                        Number of Statements    1

         Model Variables: Y T1 T2 X1 X2 X3

Parameters: A0: -20  A1: 2  B0: 1.1  B1: 0  B2: -0.02  C0: -6
           C1: -0.1  C2: 0  A2: 0  A3: 0  A4: 0.1  B3: 0
           B4: 0.3  C3: -1.2  C4: 0  D01: 5  D1: -7
           D2: -3  D3: -1  D4: 0

         Equations: Y

                   SYSNLIN Procedure

               The Equation to Estimate is:

          Y = F( A0(1), A1, B0, B1, B2, C0, C1, C2 )

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```

SYSNLIN Procedure
OLS Estimation

OLS Estimation Summary

Dataset Option	Dataset
DATA=	SHEN

Parameters Estimated	8

Minimization Summary

Method	MARQUARDT
Iterations	1

Final Convergence Criteria

R	0.00087836
PPC(C0)	0.001841
RPC(B1)	13951122
Object	0.99999607
Trace(S)	0.00196953
Objective Value	0.00196601
Lambda	1E-7

Observations Processed

Read	4480
Solved	4480

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SYSNLIN Procedure
OLS Estimation

Nonlinear OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
Y	8	4472	8.80773	0.0019695	0.04438	0.9683	0.9682

Nonlinear OLS Parameter Estimates

Parameter	Estimate	Approx. Std Err	'T' Ratio	Approx. Prob>|T|
A0	2.298739	0.01100	208.89	0.0
A1	0.239270	0.02093	11.43	0.0001
B0	-4.187422	0.11891	-35.22	0.0001
B1	13.951122	0.34766	40.13	0.0001
B2	12.727442	0.18779	67.78	0.0
C0	3.331751	0.12464	26.73	0.0001
C1	-14.312558	0.35277	-40.57	0.0001
C2	-12.820772	0.23957	-53.52	0.0

Number of Observations		Statistics for System	
Used	4480	Objective	0.001966
Missing	0	Objective*N	8.8077

SAS programing input for E_{12} prediction
$t_1 + 0.2t_2 > 7$in.

```
options ps=60;
data shen;
infile 'c:\fwd40\fwd40-2.dat';
input E1 E2 E3 E4 t1 t2 t3 D0 D10 D15 D00 D8 D12 D20 D24 D36
D60;
E12=(E1*t1+E2*t2)/(t1+t2);
y=log10(E12);
x1=D0-0.982*D12;
proc sysnlin method=marquardt;
endo y;
exo t1 t2 x1 x2 x3;
y=(A0+A1*log10(t1+A2*t2))
+(B0+B1/t1)*log10(x1);
fit y start=(A0 -20 A1 2 A2 0 A3 0.0 A4 0.1
B0 1.1 B1 0 B2 -0.02 B3 0 B4 0.3
C0 -6.0 C1 -0.1 C2 0.0 C3 -1.2 C4 0);
Parms A0 A1 A2 A3 B0 B1 B2 B3 C0 C1 C2 C3 C4;
run;
```

SAS programing output for E_{12} prediction
$t_1 + 0.2t_2 > 7$in.

```
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                              SYSNLIN Procedure

                                Model Summary

                          Model Variables          6
                            Endogenous             1
                            Exogenous              5
                          Parameters              24
                          Equations                1

                          Number of Statements     1

          Model Variables: Y T1 T2 X1 X2 X3

Parameters: A0: -20  A1: 2  A2: 0  B0: 1.1  B1: 0  A3: 0
            A4: 0.1  B2: -0.02  B3: 0  B4: 0.3  C0: -6
            C1: -0.1  C2: 0  C3: -1.2  C4: 0

          Equations: Y

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                       SYSNLIN Procedure

                   The Equation to Estimate is:

                   Y = F( A0(1), A1, A2, B0, B1 )

                       SYSNLIN Procedure
                         OLS Estimation

                     OLS Estimation Summary
```

Dataset Option	Dataset
DATA=	SHEN

Parameters Estimated 5

Minimization Summary

Method	MARQUARDT
Iterations	6

Final Convergence	Criteria
R	0.00009137
PPC(B1)	0.000273
RPC(B1)	0.004457
Object	0.00001498
Trace(S)	0.00184575
Objective Value	0.00184479
Lambda	1E-10

Observations Processed

Read	9632
Solved	9632

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SYSNLIN Procedure
OLS Estimation

Nonlinear OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
Y	5	9627	17.76905	0.0018458	0.04296	0.9703	0.9703

Nonlinear OLS Parameter Estimates

Parameter	Estimate	Approx. Std Err	'T' Ratio	Approx. Prob>|T|
A0	3.549438	0.0063624	557.87	0.0
A1	-0.812719	0.0057001	-142.58	0.0
A2	0.634771	0.0070718	89.76	0.0
B0	-0.863772	0.0063678	-135.65	0.0
B1	-0.701636	0.05485	-12.79	0.0001

Number of Observations	
Used	9632
Missing	0

Statistics for System	
Objective	0.001845
Objective*N	17.7691

SAS programing input for E_4 prediction

```
options ps=60;
data shen;
infile 'c:\fwd40\fwd40-e4.dat';
input E4 t1 t2 t3 D60 D72;
proc sysnlin method=marquardt;
endo E4;
exo D60;
E4=(A0)*D60**(B0);
fit E4 start=(A0 -20 B0 -2);
Parms A0 B0;
run;
```

SAS programing output for E_4 prediction

```
        SAS    23:48 Tuesday, August 18, 1992   1

                   SYSNLIN Procedure

                     Model Summary

               Model Variables          2
                 Endogenous             1
                 Exogenous              1
               Parameters               2
               Equations                1

               Number of Statements     1

          Model Variables: E4 D60

          Parameters: A0: -20  B0: -2

          Equations: E4

           SAS    23:48 Tuesday, August 18, 1992   2

                   SYSNLIN Procedure

             The Equation to Estimate is:

                   E4 = F( A0, B0 )

           SAS    23:48 Tuesday, August 18, 1992   3

                   SYSNLIN Procedure
                     OLS Estimation

                 OLS Estimation Summary

              Dataset Option        Dataset
              DATA=                    SHEN

              Parameters Estimated       2

                 Minimization Summary
              Method              MARQUARDT
```

Iterations	5

Final Convergence	Criteria
R	0.0003964
PPC(B0)	0.000027
RPC(B0)	0.003498
Object	0.00288756
Trace(S)	0.5450625
Objective Value	0.54498646
Lambda	1E-10

Observations Processed	
Read	14336
Solved	14336

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SYSNLIN Procedure
OLS Estimation

Nonlinear OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
E4	2	14334	7813	0.54506	0.73828	0.9966	0.9966

Nonlinear OLS Parameter Estimates

Parameter	Estimate	Approx. Std Err	'T' Ratio	Approx. Prob>|T|
A0	35.013192	0.01098	3189.65	0.0
B0	-1.010100	0.0005791	-1744.28	0.0

Number of Observations		Statistics for System	
Used	14336	Objective	0.5450
Missing	0	Objective*N	7813

SAS programing input for E_4 prediction

```
options ps=60;
data shen;
infile 'c:\fwd40\fwd40-e4.dat';
input E4 t1 t2 t3 D60 D72;
proc sysnlin method=marquardt;
endo E4;
exo D72;
E4=(A0)*D72**(B0);
fit E4 start=(A0 -20 B0 -2);
Parms A0 B0;
run;
```

SAS programing output for E_4 prediction

```
            SAS    23:48 Tuesday, August 18, 1992  13

                      SYSNLIN Procedure

                        Model Summary

                  Model Variables         2
                    Endogenous            1
                    Exogenous             1
                  Parameters              2
                  Equations               1

                  Number of Statements    1

             Model Variables: E4 D72

             Parameters: A0: -20  B0: -2

             Equations: E4

         SAS    23:48 Tuesday, August 18, 1992  14

                   SYSNLIN Procedure

                  The Equation to Estimate is:

                       E4 = F( A0, B0 )

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                      SYSNLIN Procedure
                        OLS Estimation

                    OLS Estimation Summary

                   Dataset Option        Dataset
                   DATA=                    SHEN

                   Parameters Estimated       2

                       Minimization Summary
```

Method	MARQUARDT
Iterations	5
Subiterations	5
Average Subiterations	1.00

Final Convergence	Criteria
R	0.0000713
PPC(B0)	3.806E-6
RPC(B0)	0.000605
Object	0.00012875
Trace(S)	0.32724334
Objective Value	0.32719769
Lambda	1E-6

Observations Processed	
Read	14336
Solved	14336

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SYSNLIN Procedure
OLS Estimation

Nonlinear OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
E4	2	14334	4691	0.32724	0.57205	0.9979	0.9979

Nonlinear OLS Parameter Estimates

Parameter	Estimate	Approx. Std Err	'T' Ratio	Approx. Prob>|T|
A0	29.715807	0.0070226	4231.43	0.0
B0	-0.994617	0.0004436	-2242.09	0.0

Number of Observations	
Used	14336
Missing	0

Statistics for System	
Objective	0.3272
Objective*N	4691

APPENDIX C
SAS PORGRAMING FOR THE ADAPTATION OF CONVENTIONAL FWD SYSTEM

SAS programing input for E_1 prediction
(Simulating 24-inch load spacing)

```
data shen;
infile 'c:\fwd-24\fwdn-s24.dat';
input E1 E2 E3 E4 t1 t2 t3 D0 Dy10 Dy20 Dy24 Dy40 D8 D12 D24
D36
D60;
y=log10(E1);
x1=D0-Dy10;
x2=D0-D8;
proc sysnlin method=marquardt;
endo y;
exo t1 t2 x1 x2;
y=(A0+A1*log10(t1))
+(B0+B1/t1)*log10(x1)
+(C1/t1)*log10(x2);
fit y start=(A0 -20 A1 2 A2 0 A3 0.0 A4 0.1
B0 1.1 B1 0 B2 -0.02 B3 0 B4 0.3
C0 -6.0 C1 -0.1 C2 0.0 C3 -1.2 C4 0);
Parms A0 A1 A2 A3 B0 B1 B2 B3 C0 C1 C2 C3 C4 D0 D1 D2;
run;
```

SAS programing output for E_1 prediction
(Simulating 24-inch load spacing)

```
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                       SYSNLIN Procedure

                          Model Summary

                    Model Variables         5
                      Endogenous            1
                      Exogenous             4
                    Parameters             24
                    Equations               1

                    Number of Statements    1

          Model Variables: Y T1 T2 X1 X2

Parameters: A0: -20  A1: 2  B0: 1.1  B1: 0  C1: -0.1  A2: 0
            A4: 0.1  B2: -0.02  B3: 0  B4: 0.3  C0: -6  C2: 0
            C3: -1.2  C4: 0  D01: 5  D1: -7  D2: -3  D3: -1

          Equations: Y

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                       SYSNLIN Procedure

                  The Equation to Estimate is:

                  Y = F( A0(1), A1, B0, B1, C1 )
```

```
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               SYSNLIN Procedure
                 OLS Estimation

             OLS Estimation Summary

          Dataset Option       Dataset
          DATA=                   SHEN

          Parameters Estimated       5

              Minimization Summary
          Method             MARQUARDT
          Iterations                 1

          Final  Convergence  Criteria
          R                  0.0008932
          PPC(A1)             0.001674
          RPC(B1)             19133245
          Object            0.99999424
          Trace(S)          0.00257842
          Objective Value   0.00251901
          Lambda                  1E-7

             Observations Processed
          Read                     217
          Solved                   217

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               SYSNLIN Procedure
                 OLS Estimation

      Nonlinear OLS Summary of Residual Errors

        DF    DF
Equation  Model Error   SSE    MSE   Root MSE  R-Square  Adj R-Sq

Y      5   212 0.54663  0.0025784  0.05078  0.9676    0.9670

         Nonlinear OLS Parameter Estimates

                              Approx.       'T'   Approx.
     Parameter    Estimate     Std Err      Ratio  Prob>|T|

     A0            1.893348    0.07606      24.89   0.0001
     A1            0.555398    0.07182       7.73   0.0001
     B0           -0.796624    0.03866     -20.60   0.0001
     B1          -19.133245    0.61346     -31.19   0.0001
     C1           17.479079    0.56484      30.95   0.0001

        Number of Observations        Statistics for System
        Used                 217      Objective     0.002519
        Missing                0      Objective*N     0.5466
```

SAS programing input for E_{12} prediction
(Simulating 24-inch load spacing)

```
data shen;
infile 'c:\fwd-24\fwdn-s24.dat';
input E1 E2 E3 E4 t1 t2 t3 D0 Dy10 Dy20 Dy24 Dy40 D8 D12 D20
D36
D60;
E12=(E1*t1+E2*t2)/(t1+t2);
y=log10(E12);
x1=(D0+Dy24)/2-0.985*D8;
proc sysnlin method=marquardt;
endo y;
exo t1 t2 x1;
y=(A0+A1*log10(t1)+A2*log10(t2))
+(B0)*log10(x1);
fit y start=(A0 -20 A1 2 A2 0 A3 0.0 A4 0.1
B0 1.1 B1 0 B2 -0.02 B3 0 B4 0.3);
Parms A0 A1 A2 A3 B0 B1 B2 B3;
run;
```

SAS programing output for E_{12} prediction
(Simulating 24-inch load spacing)

```
                SAS  17:34 Wednesday, August 26, 1992 224

                       SYSNLIN Procedure

                         Model Summary

                  Model Variables          4
                    Endogenous             1
                    Exogenous              3
                  Parameters              10
                  Equations                1

                  Number of Statements     1

          Model Variables: Y T1 T2 X1

Parameters: A0: -20  A1: 2  A2: 0  B0: 1.1  A3: 0  A4: 0.1
            B1: 0  B2: -0.02  B3: 0  B4: 0.3

          Equations: Y

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                       SYSNLIN Procedure

                 The Equation to Estimate is:

                   Y = F( A0(1), A1, A2, B0 )

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```

SYSNLIN Procedure
OLS Estimation

OLS Estimation Summary

Dataset Option	Dataset
DATA=	SHEN

Parameters Estimated 4

Minimization Summary

Method	MARQUARDT
Iterations	1

Final Convergence	Criteria
R	0.00058411
PPC(A2)	0.000694
RPC(A2)	408461.7
Object	0.99999451
Trace(S)	0.00246811
Objective Value	0.00242262
Lambda	1E-7

Observations Processed

Read	217
Solved	217

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SYSNLIN Procedure
OLS Estimation

Nonlinear OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
Y	4	213	0.52571	0.0024681	0.04968	0.9557	0.9551

Nonlinear OLS Parameter Estimates

Parameter	Estimate	Approx. Std Err	'T' Ratio	Approx. Prob>|T|
A0	3.006922	0.04083	73.64	0.0001
A1	-0.528558	0.03089	-17.11	0.0001
A2	-0.408462	0.03985	-10.25	0.0001
B0	-1.020199	0.01668	-61.15	0.0001

Number of Observations		Statistics for System	
Used	217	Objective	0.002423
Missing	0	Objective*N	0.5257

SAS programing input for E_3 prediction
(Simulating 24-inch load spacing)

```
options ps=60;
data shen;
infile 'c:\fwd-24\fwds-24.dat';
input E1 E2 E3 E4 t1 t2 t3 D0 D12 D14 D16 D20 D24 D36 D60
Dy24;
y=log10(E3);
x1=D60;
x2=(D0+D12)/2-1.15*D36;
x3=D36-D60;
proc sysnlin method=marquardt;
endo y;
exo t1 t2 x1 x2 x3;
y=(A0+A2*log10(t2))
+(B0+B1/x1+B3/t2)*log10(x2)
+(C0+C3/x1)*log10(x3)
+(D01+D1*t1+D2*t2+D3/x1)*log10(x1);
fit y start=(A0 -20 A1 2 A2 0 A3 0.0
B0 1.1 B1 0 B2 -0.02 B3 0 B4 0.3
C0 -6.0 C1 -0.1 C2 0.0 C3 -1.2
D01 5 D1 -7 D2 -3.0 D3 -1.0);
Parms A0 A1 A2 A3 B0 B1 B2 B3 C0 C1 C2 C3 D0 D1 D2;
run;
```

SAS programing output for E_3 prediction
(Simulating 24-inch load spacing)

```
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                         SYSNLIN Procedure

                           Model Summary

                     Model Variables          6
                       Endogenous             1
                       Exogenous              5
                     Parameters              24
                     Equations                1

                     Number of Statements     1

          Model Variables: Y T1 T2 X1 X2 X3

Parameters: A0: -20  A2: 0  B0: 1.1  B1: 0  B3: 0  C0: -6
            C3: -1.2  D01: 5  D1: -7  D2: -3  D3: -1  A1: 2
            A3: 0  B2: -0.02  C1: -0.1  C2: 0  D4: 0

          Equations: Y

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                        SYSNLIN Procedure

                    The Equation to Estimate is:

      Y = F( A0(1), A2, B0, B1, B3, C0, C3, D01, D1, D2, D3 )

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                        SYSNLIN Procedure
                          OLS Estimation

                      OLS Estimation Summary

                   Dataset Option        Dataset
                   DATA=                    SHEN

                   Parameters Estimated      11

                       Minimization Summary
                    Method                MARQUARDT
                    Iterations                    1

                    Final  Convergence  Criteria
                    R                   0.00075699
                    PPC(B3)               0.001872
                    RPC(B3)                5301995
                    Object               0.9999974
                    Trace(S)            0.00827884
                    Objective Value     0.00820857
```

Lambda 1E-7

Observations Processed	
Read	1296
Solved	1296

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SYSNLIN Procedure
OLS Estimation

Nonlinear OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
Y	11	1285	10.63831	0.0082788	0.09099	0.9035	0.9027

Nonlinear OLS Parameter Estimates

Parameter	Estimate	Approx. Std Err	'T' Ratio	Approx. Prob>|T|
A0	2.024532	0.06773	29.89	0.0001
A2	-1.078469	0.06154	-17.52	0.0001
B0	2.160875	0.06188	34.92	0.0001
B1	-1.620187	0.05547	-29.21	0.0001
B3	-5.301995	0.49663	-10.68	0.0001
C0	-6.025226	0.08956	-67.27	0.0
C3	2.488758	0.11082	22.46	0.0001
D01	3.670627	0.05820	63.07	0.0
D1	-0.049758	0.0028039	-17.75	0.0001
D2	-0.068613	0.0026032	-26.36	0.0001
D3	-3.099075	0.14451	-21.45	0.0001

Number of Observations	
Used	1296
Missing	0

Statistics for System	
Objective	0.008209
Objective*N	10.6383

SAS programing input for E_4 prediction
(Simulating 24-inch load spacing)

```
data shen;
infile 'c:\fwd-24\fwds-24.dat';
input E1 E2 E3 E4 t1 t2 t3 D0 D12 D14 D16 D20 D24 D36 D60
Dy24;
y=log10(E4);
x1=D60;
proc sysnlin method=marquardt;
endo y;
exo x1;
y=A0
+B0*log10(x1);
fit y start=(A0 -20 A1 2 A2 0 A3 0.0
B0 1.1 B1 0 B2 -0.02 B3 0);
Parms A0 A1 A2 A3 B0 B1 B2 B3;
run;
```

SAS programing output for E_4 prediction
(Simulating 24-inch load spacing)

```
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                      SYSNLIN Procedure

                        Model Summary

                  Model Variables         2
                    Endogenous            1
                    Exogenous             1
                  Parameters              8
                  Equations               1

                  Number of Statements    1

          Model Variables: Y X1

Parameters: A0: -20  B0: 1.1  A1: 2  A2: 0  A3: 0  B1: 0
            B2: -0.02  B3: 0

          Equations: Y

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                      SYSNLIN Procedure

                The Equation to Estimate is:

                      Y = F( A0(1), B0 )

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                       SYSNLIN Procedure
                        OLS Estimation
```

OLS Estimation Summary

Dataset Option	Dataset
DATA=	SHEN

Parameters Estimated	2

Minimization Summary

Method	MARQUARDT
Iterations	1

Final Convergence	Criteria
R	0.00017642
PPC(B0)	8.346E-6
RPC(B0)	1.922672
Object	0.99999906
Trace(S)	0.00040261
Objective Value	0.00040199
Lambda	1E-7

Observations Processed

Read	1296
Solved	1296

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SYSNLIN Procedure
OLS Estimation

Nonlinear OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
Y	2	1294	0.52098	0.0004026	0.02007	0.9965	0.9964

Nonlinear OLS Parameter Estimates

Parameter	Estimate	Approx. Std Err	'T' Ratio	Approx. Prob>|T|
A0	1.560313	0.0008791	1774.84	0.0
B0	-1.014941	0.0016838	-602.77	0.0

Number of Observations	
Used	1296
Missing	0

Statistics for System	
Objective	0.000402
Objective*N	0.5210

SAS programing input for E_1 prediction
(Simulating 40-inch load spacing)

```
data shen;
infile 'c:\fwd-24\fwdn-s24.dat';
input E1 E2 E3 E4 t1 t2 t3 D0 Dy10 Dy20 Dy24 Dy40 D8 D12 D20
D36
D60;
y=log10(E1);
x1=D0-Dy10;
x2=D0-D20;
proc sysnlin method=marquardt;
endo y;
exo t1 t2 x1 x2;
y=(A0)
+(B0+B1/t1)*log10(x1)
+(C1/t1)*log10(x2);
fit y start=(A0 -20 A1 2 A2 0 A3 0.0 A4 0.1
B0 1.1 B1 0 B2 -0.02 B3 0 B4 0.3
C0 -6.0 C1 -0.1 C2 0.0 C3 -1.2 C4 0
D01 5 D1 -7 D2 -3.0 D3 -1.0 D4 0 E0 -0.5 E1 0 E2 1);
Parms A0 A1 A2 A3 B0 B1 B2 B3 C0 C1 C2 C3 C4 D0 D1 D2;
run;
```

SAS programing output for E_1 prediction
(Simulating 40-inch load spacing)

```
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                     SYSNLIN Procedure

                       Model Summary

                 Model Variables          5
                   Endogenous             1
                   Exogenous              4
                 Parameters              24
                 Equations                1
                 Number of Statements     1

       Model Variables: Y T1 T2 X1 X2

Parameters: A0: -20  B0: 1.1  B1: 0  C1: -0.1  A1: 2  A2: 0
            B2: -0.02  B3: 0  B4: 0.3  C0: -6  C2: 0

          Equations: Y

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                     SYSNLIN Procedure

               The Equation to Estimate is:

                 Y = F( A0(1), B0, B1, C1 )
```

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SYSNLIN Procedure
OLS Estimation

OLS Estimation Summary

Dataset Option	Dataset
DATA=	SHEN

Parameters Estimated	4

Minimization Summary

Method	MARQUARDT
Iterations	1

Final Convergence	Criteria
R	0.000109
PPC(B1)	0.00005
RPC(B1)	9364031
Object	0.99999254
Trace(S)	0.00381158
Objective Value	0.00374132
Lambda	1E-7

Observations Processed

Read	217
Solved	217

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SYSNLIN Procedure
OLS Estimation

Nonlinear OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
Y	4	213	0.81187	0.0038116	0.06174	0.9519	0.9513

Nonlinear OLS Parameter Estimates

Parameter	Estimate	Approx. Std Err	'T' Ratio	Approx. Prob>|T|
A0	2.449082	0.01461	167.62	0.0001
B0	-0.860069	0.04616	-18.63	0.0001
B1	-9.364031	0.38555	-24.29	0.0001
C1	7.566191	0.22087	34.26	0.0001

Number of Observations	
Used	217
Missing	0

Statistics for System	
Objective	0.003741
Objective*N	0.8119

SAS programing input for E_{12} prediction
(Simulating 40-inch load spacing)

```
options ps=60;
data shen;
infile 'c:\fwd-24\fwdn-s24.dat';
input E1 E2 E3 E4 t1 t2 t3 D0 Dy10 Dy20 Dy24 Dy40 D8 D12 D20
D36
D60;
E12=(E1*t1+E2*t2)/(t1+t2);
y=log10(E12);
x1=(D0+Dy40)/2-D20;
x2=D0-1.03*Dy10;
proc sysnlin method=marquardt;
endo y;
exo t1 t2 x1;
y=(A0+A1*log10(t1)+A2*log10(t2))
+(B0+B1/t1)*log10(x1)
+(C1/t1)*log10(x2);
fit y start=(A0 -20 A1 2 A2 0 A3 0.0 A4 0.1
B0 1.1 B1 0 B2 -0.02 B3 0 B4 0.3 C0 1.1
C1 0 C2 -0.1);
Parms A0 A1 A2 A3 B0 B1 B2 B3 C0 C1 C2;
run;
```

SAS programing output for E_{12} prediction
(Simulating 40-inch load spacing)

```
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                        SYSNLIN Procedure

                          Model Summary

                    Model Variables        4
                      Endogenous           1
                      Exogenous            3
                    Parameters            13
                    Equations              1

                    Number of Statements   1

          Model Variables: Y T1 T2 X1

Parameters: A0: -20  A1: 2  A2: 0  B0: 1.1  B1: 0  C1: 0
           A3: 0  A4: 0.1  B2: -0.02  B3: 0  B4: 0.3  C0: 1.1
          C2: -0.1

     Equations: Y

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                        SYSNLIN Procedure

                    The Equation to Estimate is:
```

Y = F(A0(1), A1, A2, B0, B1, C1)

SYSNLIN Procedure
OLS Estimation

OLS Estimation Summary

Dataset Option	Dataset
DATA=	SHEN

Parameters Estimated	6

Minimization Summary

Method	MARQUARDT
Iterations	1

Final Convergence	Criteria
R	0.00083273
PPC(A2)	0.000669
RPC(C1)	7151238
Object	0.99999581
Trace(S)	0.00185605
Objective Value	0.00180473
Lambda	1E-7

Observations Processed

Read	217
Solved	217

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SYSNLIN Procedure
OLS Estimation

Nonlinear OLS Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj R-Sq
Y	6	211	0.39163	0.0018561	0.04308	0.9670	0.9662

Nonlinear OLS Parameter Estimates

Parameter	Estimate	Approx. Std Err	'T' Ratio	Approx. Prob>|T|
A0	3.640915	0.04200	86.69	0.0001
A1	-0.837191	0.03281	-25.51	0.0001
A2	-0.466646	0.03477	-13.42	0.0001
B0	-0.928174	0.02721	-34.11	0.0001
B1	6.437375	0.35061	18.36	0.0001
C1	-7.151238	0.28070	-25.48	0.0001

Number of Observations		Statistics for System	
Used	217	Objective	0.001805
Missing	0	Objective*N	0.3916

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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